Equatorial electron loss by double resonance with oblique and parallel intense chorus waves

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- Equatorially mirroring electron scattering by double Landau and cyclotron resonance
- Formation of 30–150 keV butterfly electron distributions by nonlinear wave-particle interactions
- Possible contribution of oblique and parallel chorus waves to energetic electron dropouts

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Equatorial electron loss by double resonance with oblique and parallel intense chorus waves

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Abstract Puzzling satellite observations of butterfly pitch angle distributions and rapid dropouts of 30–150 keV electrons are widespread in the Earth's radiation belts. Several mechanisms have been proposed to explain these observations, such as enhanced outward radial diffusion combined with magnetopause shadowing or scattering by intense magnetosonic waves, but their effectiveness is mainly limited to storm times. Moreover, the scattering of 30–150 keV electrons via cyclotron resonance with intense parallel chorus waves should be limited to particles with equatorial pitch angle smaller than 70°–75°, leaving unaffected a large portion of the population. In this paper, we investigate the possible effects of oblique whistler mode waves, noting, in particular, that Landau resonance with very oblique waves can occur up to ∼89°. We demonstrate that such very oblique chorus waves with realistic amplitudes can very efficiently nonlinearly transport nearly equatorially mirroring electrons toward smaller pitch angles where nonlinear scattering (phase bunching) via cyclotron resonance with quasi-parallel waves can take over and quickly send them to much lower pitch angles <40°. The proposed double resonance mechanism could therefore explain the formation of butterfly pitch angle distributions as well as contribute to some fast dropouts of 30–150 keV electrons occurring during moderate geomagnetic disturbances at $L = 4$–6. Since 30–150 keV electrons represent a seed population for a further acceleration to relativistic energies by intense parallel chorus waves during storms or substorms, the proposed mechanism may have important consequences on the dynamics of 100 keV to MeV electron fluxes in the radiation belts.

1. Introduction

The existence in the inner magnetosphere of rapid dropouts of almost whole populations of energetic electrons (with energies $E \sim 25$–200 keV) is now well known [e.g., see *Millan and Thorne*, 2007; *Morley et al.*, 2010; *Turner et al.*, 2012, 2014a, 2014b; *Albert*, 2014; *Gao et al.*, 2015; *Hwang et al.*, 2015, and references therein], as well as the disappearance in various situations of nearly equatorially mirroring energetic electrons, leading to the formation of so-called butterfly distributions in equatorial pitch angle α_0 [e.g., see *Gannon et al.*, 2007; *Gu et al.*, 2011; *Zhao et al.*, 2014]. Such rapid variations of the distribution of energetic electrons may have profound and direct mitigating consequences on internal charging hazards inside various spacecrafts [e.g., see *Mulligan-Skov et al.*, 2015; *Tian et al.*, 2015]. They may also strongly impact the flux levels of MeV "satellite killer" electrons in the following periods, since it is this seed population which later gets accelerated by intense chorus waves to relativistic energies in the heart of the radiation belts [e.g., see *Horne et al.*, 2005; *Thorne*, 2010; *Horne et al.*, 2013; *Turner et al.*, 2014b; *Jaynes et al.*, 2015; *Mourenas et al.*, 2015a].

What can explain such observations? And more specifically, how to solve the problem of nearly equatorially mirroring energetic electron scattering and loss? During storm periods of strong magnetic field perturbations and/or high $L > 6$ (with L McIlwain's number), drift shell splitting and enhanced outward radial diffusion up to the magnetopause can certainly operate [*Sibeck et al.*, 1987; *Shprits et al.*, 2008a; *Kim et al.*, 2010; *Albert*, 2014; *Turner et al.*, 2014b]. Magnetopause shadowing may progressively entail losses at lower L, down to $L \sim 4$, due to strong radial diffusion by ultralow frequency (ULF) waves in the presence of a steeply decreasing phase space density toward higher L [*Elkington et al.*, 2003; *Shprits et al.*, 2008a; *Ukhorskiy et al.*, 2015]. However, during nonstorm times with weak magnetic disturbances at low $L<6$, other mechanisms should likely take over [*Morley et al.*, 2010; *Turner et al.*, 2012; *Albert*, 2014]. Worthy candidates are electromagnetic ion cyclotron (EMIC) waves, fast magnetosonic waves, or chorus whistler mode waves, which may transport particles to smaller pitch angles via resonant interactions, ultimately leading to their precipitation into the atmosphere [e.g., *Shprits et al.*, 2008b; *Thorne*, 2010]. Resonant wave-particle interaction requires that the wave angular frequency $\omega = 2\pi f$ (with f its frequency) and wave number component $k_\parallel$ parallel to the geomagnetic field satisfy the resonance condition $\gamma\omega + n\Omega_{ce} = k_\parallel p_\parallel$ with electrons of parallel momentum $p_\parallel$, where $n = 0$ for Landau resonance and $n = -1$ for cylotron resonance, Ω_{ce} is the electron gyrofrequency, and γ the relativistic factor [*Shklyar and Matsumoto*, 2009]. However, EMIC waves cannot interact via cyclotron resonance with $E < 300$ keV electrons of equatorial pitch angles $\alpha_0 > 70°$ [e.g., see *Summers and Thorne*, 2003; *Kersten et al.*, 2014].

Fast magnetosonic waves may interact resonantly (through Landau resonance) with such high pitch angle electrons when the electron plasma to cyclotron frequency ratio Ω_{pe}/Ω_{ce} is larger than 8 [*Mourenas et al.*, 2013], as well as nonresonantly under certain circumstances [*Bortnik and Thorne*, 2010; *Artemyev et al.*, 2015b]. In the presence of a usually decreasing phase space density of electrons toward higher E, these waves should mainly scatter high pitch angle ($\alpha_0 > 70°$) electrons toward higher energy and, by conservation of their first adiabatic invariant (in case of Landau resonant interaction), toward lower pitch angles. Due to a maximum of scattering around $E \sim 200-500$ keV when $\Omega_{pe}/\Omega_{ce} > 4.5$ [*Mourenas et al.*, 2013], this kind of process might lead to the formation of butterfly distributions in this particular energy range. Nevertheless, it becomes much less efficient at $E \leq 100$ keV, in general, especially for electrons at $\alpha_0 \geq 78°$ [*Mourenas et al.*, 2013; *Artemyev et al.*, 2015b]. Bounce resonance with these waves can be potentially important, as it could succeed in decreasing electron pitch angles from about 90° [*Roberts and Schulz*, 1968; *Schulz and Lanzerotti*, 1974; *Shprits*, 2009; *Chen et al.*, 2015; *Li et al.*, 2015]. But fast magnetosonic waves, which can reach high amplitudes close to the equator, have much lower occurrences and much smaller intensities during nonstorm times with $D_{st} > -30$ nT and moderate $K_p \leq 3$ periods [*Nemec et al.*, 2015]. During such moderately disturbed periods, therefore, other mechanisms should be invoked—involving, for instance, whistler mode waves. In particular, oblique lower band chorus waves with large parallel electric field components and refractive index values $N \leq 200-300$ have been frequently observed, even during moderately disturbed periods, in recent satellite statistics from Cluster [*Santolík et al.*, 2009; *Agapitov et al.*, 2013; *Mourenas et al.*, 2014; *Artemyev et al.*, 2015a], Time History of Events and Macroscale Interactions during Substorms (THEMIS) [*Li et al.*, 2013; *Agapitov et al.*, 2014a; *Taubenschuss et al.*, 2014], and the Van Allen Probes [*Li et al.*, 2014; *Mourenas et al.*, 2015b], calling our attention to these specific waves.

In the second section, we discuss the different characteristics of Landau and cyclotron resonances with parallel and oblique whistler mode waves, in particular, the corresponding pitch angle and energy ranges of each resonance. Next, we examine nonlinear effects of high-amplitude whistler mode waves on 30–150 keV electrons in the inhomogeneous geomagnetic field. Contrary to small-amplitude waves which produce small and stochastic variations of electron energy and pitch angle, corresponding to a slow quasi-linear diffusion over timescales longer than a day [*Mourenas et al.*, 2012, 2015a], intense waves can lead to strong, fast, and deterministic changes in particle energy and pitch angle (e.g., see the review by *Shklyar and Matsumoto* [2009]). When a wave is sufficiently intense, the effect of its electromagnetic field can compensate the effect of the mirror force on particles moving in the inhomogeneous magnetic field. The competition of these forces generates a local potential well in the phase space where particles can be trapped and oscillate at the trapping frequency around an equilibrium position. Electrons which briefly pass through this equilibrium (for a time about the trapping period) experience nonlinear scattering, also called "phase bunching" [e.g., see *Albert*, 1993, 2002; *Artemyev et al.*, 2014b]. Alternatively, particles initially untrapped can become trapped for a time much longer than the trapping period, if the area of the potential well increases along the particle trajectory, corresponding either to an increasing wave amplitude or a decreasing magnetic field inhomogeneity [*Albert*, 2002; *Omura et al.*, 2009; *Shklyar and Matsumoto*, 2009; *Artemyev et al.*, 2014b]. Changes in energy and pitch angle occurring during a given nonlinear trapping event are generally much larger than for a brief nonlinear scattering event, but the probability of trapping is usually smaller than that of nonlinear scattering [*Albert*, 2002; *Artemyev et al.*, 2014b, 2015c]. In the third section, we examine nonlinear effects related to electron trapping which can increase the pitch angle range potentially affected by Landau and cyclotron resonances with parallel or oblique chorus waves. Then, section 4 is devoted to an investigation of the possible consequences of electron nonlinear interactions with very oblique and parallel chorus waves. In particular, we shall provide useful estimates of wave amplitude and obliquity levels required to potentially obtain a strong and fast depletion of 30–150 keV nearly equatorially mirroring electrons. These results will finally be discussed in connection with recent observations from the Van Allen Probes.

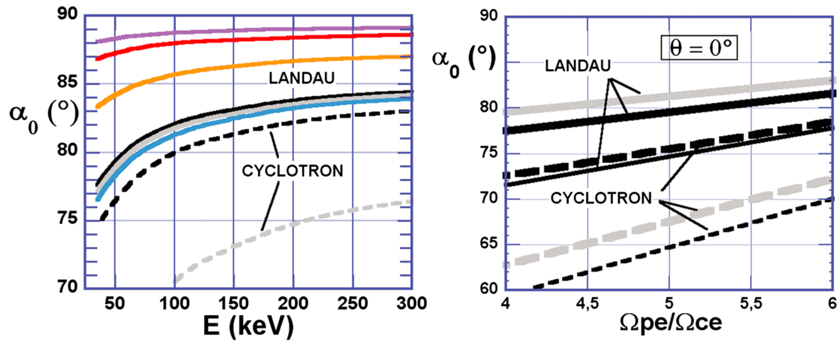

Figure 1. (left) Equatorial pitch angle α_0 from equation (1) for Landau resonance with lower band chorus whistler mode waves, as a function of electron energy E, for $\omega/\Omega_{ce} = 0.3$ and $\Omega_{pe}/\Omega_{ce} = 6$ (i.e., typically $L \sim 5$–6.5). The different curves correspond to θ= 0° (solid black line), θ= 30° (solid grey line), $\theta = \theta_g = \arccos(2\omega/\Omega_{ce})$ (blue line), $\theta = \theta_r - 4.6°$ with $\theta_r = \arccos(\omega/\Omega_{ce})$ (orange line), $\theta = \theta_r - 1°$ (red line), and $\theta \simeq \theta_r - 0.18°$ (purple line). The last four cases correspond, respectively, to wave refractive index N = 20, 40, 85, and 200. The value of α_0 at fundamental cyclotron resonance is plotted for $\theta = \theta_r - 4.6°$ (dashed black line) and θ= 30° (dashed grey line). (right) Equatorial pitch angle α_0 for Landau (solid lines) and cyclotron (dashed lines) resonances with parallel lower band chorus waves, as a function of the equatorial ratio Ω_{pe}/Ω_{ce}, for E = 100 keV and $\omega/\Omega_{ce} = 0.25$ (thick grey lines) or $\omega/\Omega_{ce} = 0.35$ (thick black lines) and for E = 40 keV, $\omega/\Omega_{ce} = 0.35$ (thin black lines).

2. Landau and Cyclotron Resonances With Whistler Mode Waves

Whistler mode waves are one possible candidate for the resonant scattering of nearly equatorially mirroring electrons. Landau resonance (also called Cerenkov resonance) occurs when the parallel component of the electron velocity is equal to the parallel component of the wave phase velocity. It can occur up to high equatorial pitch angles α_0, with a maximum value α_{0L} given by

$$\cos\alpha_{0L} = \frac{\gamma m_0 c}{p} \frac{\sqrt{\omega\left(\Omega_{ce}\cos\theta - \omega\right)}}{\Omega_{pe}\cos\theta} \tag{1}$$

where θ is the wave normal angle, m_0 the electron rest mass ($m_0 = m/\gamma$ with m its full mass), and c the velocity of light. Note that the approximate whistler mode dispersion relation used in equation (1), as well as in the remainder of this paper, is valid when $\Omega_{pe}/\Omega_{ce} > \sin\theta$ and $\Omega_{pe}/\Omega_{ce} \gg \omega/\Omega_{pe}$ locally at resonance [e.g., *Artemyev et al.*, 2013a]. Quasi-parallel whistler mode waves (with θ <30°) in the lower band chorus ($0.1\Omega_{ce} \leq \omega \leq 0.5\Omega_{ce}$) frequency range are rather ubiquitous outside the plasmasphere and among the most intense electromagnetic waves present in the inner magnetosphere on average [e.g., see *Meredith et al.*, 2012; *Agapitov et al.*, 2013; *Li et al.*, 2013, and references therein]. Nevertheless, when considering quasi-parallel waves and low electron energy $E < 200$ keV at $L \sim 6$ (with $\Omega_{pe}/\Omega_{ce} \sim 6$), only waves of sufficiently low-frequency $\omega < 0.3\Omega_{ce}$ can interact via Landau resonance with particles at $\alpha_0 > 83°$ (see Figure 1). In addition, the upper bound on α_0 given by equation (1) for Landau resonance decreases as E or Ω_{pe}/Ω_{ce} go down.

The maximum equatorial pitch angle α_{0C} where cyclotron resonance is available can be expressed as a function of the maximum equatorial pitch angle α_{0L} for Landau resonance under the form

$$\cos\alpha_{0C} = \cos\alpha_{0L}\left(\frac{\Omega_{ce}}{\gamma\omega} - 1\right), \tag{2}$$

corresponding to sensibly smaller α_0 than for Landau resonance when considering low-frequency parallel or oblique waves such that $2\omega < \Omega_{ce}/\gamma$ (Figure 1) [see also *Albert*, 2002; *Mourenas et al.*, 2012; *Artemyev et al.*, 2012]. Thus, Landau resonance looks more promising than cyclotron resonance for such low-frequency waves. However, Landau and cyclotron resonances with quasi-parallel chorus waves are not available all the way up to $\alpha_0 \simeq 90°$ for such relatively low energy particles, leaving unaffected the part of their population situated above $\alpha_0 \sim 75°$–80° [see also *Horne et al.*, 2005; *Mourenas et al.*, 2014]. For a typical initial shape $f(\alpha_0) \sim \sin^{3/2}\alpha_0$ of the electron distribution (as near the beginning of the period of the Van Allen Probe measurements considered in section 4), the unaffected electrons at $\alpha_0 \geq 75°$–80° represent an important fraction (~25–40%) of the total population. Moreover, it is plain to see in Figure 1 that contrary to parallel waves, very oblique lower band chorus waves can reach Landau resonance (but not cyclotron resonance) with electrons very close to

$\alpha_0 = 90°$, provided that they propagate at highly oblique angles, close enough to their resonance cone angle $\theta_r = \arccos(\omega/\Omega_{ce})$. Thus, such very oblique whistler mode waves could really impact nearly all 30–200 keV electrons at $\alpha_0 > 75°$.

But one question remains: if oblique upper band chorus waves (with $\omega > \Omega_{ce}/2$) with mean frequency $\omega_{\rm UB}$ are present together with (generally much more intense) oblique lower band chorus waves with mean frequency $\omega_{\rm LB}$, can these upper band waves reach resonance with electrons at higher α_0 than lower band waves? Let us assume that both waves have the same value of $q = \cos\theta/\cos\theta_r \sim 1.01–2$ and consequently the same refractive index N *normalized* to their respective maximum refractive index $N_{\max} \propto 1/\omega$ allowed by Landau damping from ∼100–500 eV suprathermal electrons and thermal effects [*Mourenas et al.*, 2014; *Li et al.*, 2014]. Then, Landau resonance occurs at the same α_0 for both kinds of waves, but upper band waves can reach cyclotron resonance at a sensibly higher α_0 than Landau resonance with lower band waves when $\gamma \geq \Omega_{ce}/(\omega_{\rm UB} + 0.7\omega_{\rm LB})$. Taking typical values $\omega_{\rm UB}/\Omega_{ce}$∼0.58 and $\omega_{\rm LB}/\Omega_{ce} \sim 0.3$, it corresponds to the range $E >$130 keV. For $\omega_{\rm UB}$ close enough to Ω_{ce}/γ, cyclotron resonance even becomes available up to $\alpha_0 \simeq 90°$ regardless of the value of θ. Nevertheless, it should mainly concern a very narrow domain of pitch angles at $\alpha_0 \simeq 88°–90°$ for $E > 130$ keV. Moreover, upper band waves are usually significantly less intense than lower band waves [*Meredith et al.*, 2012]. Thus, we shall focus below on lower band chorus waves.

3. Nonlinear Effects of Intense Chorus Waves

3.1. Quasi-Parallel Chorus Waves

For quasi-parallel waves of large enough amplitudes (typically magnetic amplitudes $B_{w,\rm total} > 100$ pT), nonlinear effects enter into play. But electron trapping via cyclotron resonance can only increase pitch angles, whereas nonlinear cyclotron scattering (i.e., phase bunching) increases pitch angles too at the highest α_0 values where cyclotron resonance remains available and decreases pitch angles only at smaller initial $\alpha_0 < 80°$ for $E \leq 150$ keV and $\omega/\Omega_{ce} \sim 0.3$ at $L \sim 5-6$ [e.g., see *Albert*, 2002, Figure 1].

However, nonlinear wave-particle interaction has one additional important property: for high-amplitude waves, the wavefield starts determining the effective resonance width, which otherwise depends only on wave dispersion in the frame of quasi-linear theory [e.g., *Karpman*, 1974]. The full width of the resonance can be defined as the size of the region in phase space filled by closed trajectories, i.e., trajectories oscillating around the resonance condition at a frequency $\Omega_{\rm tr}$ (called the trapping frequency) and, thus, spending a long time interval within resonance with the wave (e.g., see explanations in *Shklyar and Matsumoto* [2009]). The phase space domain of closed trajectories is surrounded by a separatrix. As a result, even particles with parameters not satisfying the resonance condition can be located within a distance (in pitch angle/energy space) from this resonance smaller than the effective resonance width. In this case, formally nonresonant particles can resonantly interact with the waves. But actual trapping can occur only when an additional condition is satisfied: the phase space domain surrounded by the separatrix should increase along the particle trajectory, allowing the particles to get trapped (see section 4). Let us consider here the domain of possible resonance. The extended width $\Delta\omega$ around the resonance is roughly given by the trapping frequency Ω_{tr} (see, e.g., *Artemyev et al.* [2014a] and supplementary materials to this paper). For parallel whistler mode waves interacting with electrons at $\alpha_0 \sim 90°$ with p quasi-constant, one has $\Delta\alpha_0 \sim \Delta\cos\alpha_0 \simeq \Delta p_\parallel/p$ and thus an extended trapping region around the resonance given by

$$\Delta\alpha_0 \approx 2\sqrt{\frac{m_0 c}{p}\frac{\Omega_{ce}}{\Omega_{pe}}\frac{B_{w,\rm total}}{B_0}\left(\frac{\Omega_{ce}}{\omega}-1\right)^{1/2}} \tag{3}$$

where B_0 is the equatorial geomagnetic field strength. For $L \sim 6$, $\omega/\Omega_{ce} \sim 0.3$, $E \sim 50$ keV and $B_{w,\rm total} \sim 100$ pT, one gets $\Delta\alpha_0 \sim 2.5°$. The region where cyclotron resonance can be reached is therefore increased by 2.5° toward 90° as compared with estimates for low-amplitude waves. Actually, the *time-averaged* level of the waves needs not be extremely elevated. The presence of a few isolated bursts of high-amplitude parallel waves could be sufficient to allow equatorially mirroring electrons to escape from the pitch angle domain nearby 90°. For parallel waves and cyclotron resonance, equations (2) and (3) show that it requires

$$\frac{B_{w,\rm total}}{B_0} > \frac{(\Omega_{ce}-\gamma\omega)^2}{4\Omega_{ce}\Omega_{pe}}\sqrt{\frac{\Omega_{ce}}{\omega}-1} \tag{4}$$

Typically, isolated wave bursts have a duration Δt corresponding to a fraction of the bounce period of electrons [*Agapitov et al.*, 2014a; *Santolík et al.*, 2014], while the trapping frequency $\Omega_{tr} \gg 2\pi/\Delta t$. Note also that

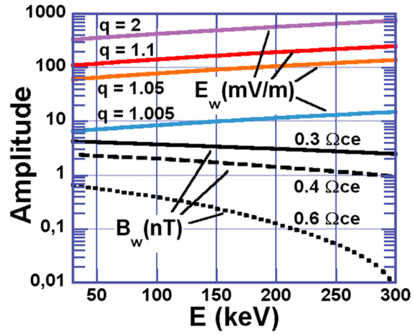

Figure 2. Minimum magnetic field amplitude $B_{w,\text{total}}$ (in nT) of parallel whistler mode waves from equation (4) as a function of electron energy E, such that the trapping width around cyclotron resonance fills the pitch angle range up to 90° (solid black line for $\omega = 0.3\Omega_{ce}$, dashed black line for $\omega = 0.4\Omega_{ce}$, and dotted black line for $\omega = 0.6\Omega_{ce}$). The minimum electric field amplitude $E_{w,\text{total}}$ (in mV/m) of oblique whistler mode waves from equation (9) is also plotted (colored lines), corresponding to a trapping width around Landau resonance filling the pitch angle range up to 90°, for different wave obliquities $q = \cos\theta / \cos\theta_r = 1.005$ (blue curve), $= 1.05$ (orange curve)), $= 1.1$ (red curve), $= 2$ (purple curve), and $\omega = 0.3\Omega_{ce}$. In both cases, we take $\Omega_{pe}/\Omega_{ce} = 6$ at $L = 6$.

geomagnetic field and plasma inhomogeneities can be neglected for first-order estimates when considering electrons with $\alpha_0 > 70°$ which are mirroring at magnetic latitudes $\lambda < 10°$.

The corresponding necessary wave amplitudes are plotted in Figure 2 for parallel chorus waves and $\Omega_{pe}/\Omega_{ce} = 6$ at $L \sim 6$. Typically, lower band chorus amplitudes larger than 4 nT are required for $\omega/\Omega_{ce} = 0.3$ and $E < 100$ keV electrons. Such very intense waves have seldom (if ever) been observed [*Santolík et al.*, 2014]. Upper band chorus waves at $\omega/\Omega_{ce} = 0.6$ with realistic amplitudes $\approx 10-50$ pT could succeed, but only over a narrow range of electron energies $E \sim 300-400$ keV such that $\Omega_{ce}/\gamma \sim \omega$. Consequently, cyclotron resonance with quasi-parallel waves should generally leave most of the electron distribution close to $\alpha_0 = 90°$ nearly unchanged. Nonlinear Landau resonant interaction with quasi-parallel waves requires sufficiently high parallel wave electric fields, i.e., the wave electric field amplitude should be larger than the effective force induced by the geomagnetic field inhomogeneity [*Shklyar and Matsumoto*, 2009; *Agapitov et al.*, 2014a]. For pitch angles $\alpha_0 \geq 80°$ at $L \sim 4-6$, the minimum value of the parallel electric field needed to get nonlinear effects via Landau resonance is larger than 10 mV/m [*Agapitov et al.*, 2014a]. For quasi-parallel waves with $\theta < 30°$, it corresponds to very high magnetic amplitudes larger than 1.5 nT. Therefore, most of the observed quasi-parallel waves cannot remove by themselves the large amounts of energetic 30–150 keV electrons trapped in the upper range $\alpha_0 \geq 80°$.

Nonetheless, it is worth emphasizing that at lower $\alpha_0 < 70°-75°$, nonlinear cyclotron scattering by parallel chorus waves of realistic amplitudes can efficiently decelerate energetic electrons and strongly reduce their pitch angles [*Albert*, 2002; *Artemyev et al.*, 2015c], possibly allowing in the end their precipitation in the atmosphere at low energy $E \sim 20-100$ keV. Nonlinear cyclotron scattering by intense parallel chorus waves could therefore play an important role in electron dropouts—provided that some additional mechanism first succeeds in transporting abundant electrons from higher pitch angles toward this lower range $\alpha_0 < 70°-80°$. But what mechanism? With which waves? Possible answers are examined below.

3.2. Oblique Chorus Waves

3.2.1. Oblique Wavefields

Let us consider now oblique lower band chorus waves with $\theta > 30°$. An important component $E_{w\parallel}$ of their total electric field $E_{w,\text{total}}$ is directed along the background magnetic field line, favoring wave-particle interactions via Landau resonance:

$$\frac{E_{w\parallel}^2}{E_{w,\text{total}}^2} = \frac{\cos^2\theta}{(\epsilon^2+1)^2 + 2\epsilon^4\cot^2\theta} \tag{5}$$

with $\epsilon = \Omega_{pe}/(kc) = (\Omega_{ce}\cos\theta/\omega - 1)^{1/2}$ [*Verkhoglyadova et al.*, 2010]. It gives a ratio $E_{w\parallel}/E_{w,\text{total}} \sim 0.25$ for $\theta = 45°$ and $\omega = 0.3\Omega_{ce}$, reaching

$$\frac{E_{w\parallel}}{E_{w,\text{total}}} \approx \frac{\omega}{\Omega_{ce}} \tag{6}$$

for θ lying between the Gendrin angle $\theta_g \simeq \arccos(2\omega/\Omega_{ce})$ and the resonance cone angle $\theta_r \simeq \arccos(\omega/\Omega_{ce})$. At the same time, the wave electric to magnetic field ratio increases as θ increases, from $E_{w,\text{total}}/(cB_{w,\text{total}}) \sim (\Omega_{ce}/\Omega_{pe})/(\epsilon + \epsilon^{-1}) \approx \Omega_{ce}/2\Omega_{pe}$ at $\theta < 30°, \theta_g$, until it becomes

$$\frac{E_{w,\text{total}}}{cB_{w,\text{total}}} \approx \frac{\Omega_{ce}}{2^{1/2}\Omega_{pe}\epsilon} \quad \text{for } \epsilon \ll 1. \tag{7}$$

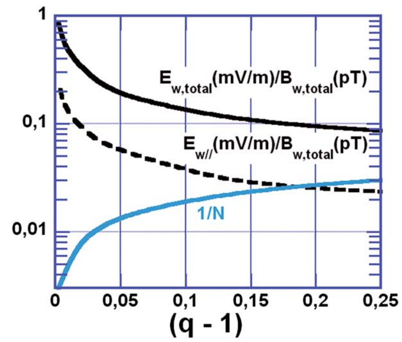

Figure 3. Ratios of the parallel component $E_{w\parallel}$ (dashed lines) and total amplitude (solid lines) of the electric field $E_{w,\text{total}}$ (in mV/m) of very oblique whistler mode waves (such that $\cos\theta < 1.5\omega/\Omega_{ce}$) over their total magnetic field amplitude $B_{w,\text{total}}$ (in pT), from equations (6) and (7) as a function of parameter $\epsilon^2 = q - 1 = \cos\theta/\cos\theta_r - 1$, taking $\omega/\Omega_{ce} = 0.3$ and $\Omega_{pe}/\Omega_{ce} \simeq 5$ at $L \sim 5-6$. The corresponding value of $1/N = \omega/kc$ is also plotted (blue curve).

As a result, very oblique whistler mode waves become quasi-electrostatic with a large $E_{w\parallel}$ and a comparatively small $B_{w,\text{total}}$ (see Figure 3). It is an important point when examining satellite data: since the energy of these very oblique waves is mostly stored under the form of electrostatic energy in $E^2_{w,\text{total}}$, one should better track their electric field amplitudes instead of their magnetic amplitudes as usual [*Artemyev et al.*, 2015a].

3.2.2. Nonlinear Interactions With Oblique Waves

Equation (1) implies that Landau resonance can exist up to nearly $\alpha_0 = 90°$ when θ gets close enough to the resonance cone angle θ_r (see Figure 1). Since the first adiabatic invariant $M = p^2 \sin^2\alpha_0/(2m_0B_0)$ is preserved during Landau resonance interaction [e.g., *Shklyar and Matsumoto*, 2009], electron accelerations via nonlinear scattering or trapping by intense oblique waves are all accompanied by a simultaneous reduction of particle pitch angles. Thus, Landau resonance with very oblique whistler mode waves could really explain the transport of high α_0 electrons of 30–150 keV toward lower pitch angles [*Artemyev et al.*, 2012, 2013b, 2014a; *Agapitov et al.*, 2015], where nonlinear scattering via cyclotron resonance with intense quasi-parallel waves should be able to rapidly send them to much smaller pitch angles closer to the loss cone—while simultaneously reducing their energy. If one further allows for some important quasi-linear pitch angle scattering by lower amplitude ~50–100 pT parallel chorus waves on the dawnside [*Meredith et al.*, 2012; *Agapitov et al.*, 2013] at later magnetic local time (MLT) or universal time (UT), then whole populations of 30–150 keV electrons could get precipitated into the atmosphere in less than 1 h (see electron lifetime estimates in *Mourenas et al.* [2012], *Artemyev et al.* [2013a], and *Mourenas et al.* [2014]) or disappear from their initial energy channel, ultimately resulting in electron flux dropouts at 30–150 keV. However, if the initial transport from high to moderate equatorial pitch angles via nonlinear scattering and trapping by intense waves is not effective enough (due to not enough bursts of intense enough waves at close enough time intervals), then a significant portion of the electron population at $\alpha_0 > 50°$ should remain in the radiation belts where such particles should mainly get accelerated via quasi-linear diffusion by the same ~50–100 pT parallel chorus waves [e.g., see *Horne and Thorne*, 2003; *Mourenas et al.*, 2014, and references therein].

For very oblique waves near θ_r, similarly as for parallel waves before, particle trapping in intense wavefields can increase the phase space domain where resonance may become available, giving now

$$\Delta\alpha_0 \approx \frac{2m_0c}{p}\sqrt{\frac{\Omega_{ce}}{\Omega_{pe}}\frac{E_{w,\text{total}}}{cB_0}\left(\frac{\Omega_{ce}}{\omega}-1\right)^{1/2}} \tag{8}$$

For the same parameters as before ($L \sim 6$, $\omega/\Omega_{ce} \sim 0.3$, $E \sim 50$ keV) but for $E_{w,\text{total}} \sim 50$ mV/m, one gets now $\Delta\alpha_0 \sim 4.5°$. Thus, very oblique wave trapping can increase the pitch angle region wherein resonance can be reached by more than 4° toward 90° as compared with low amplitude waves—filling significantly (or even totally) the no-resonance zone. Moreover, the *average* level of the waves must not necessarily be very high: a few isolated bursts of high amplitude very oblique chorus waves during a given time period could suffice to allow transporting electrons away from $\alpha_0 \approx 90°$. Using equations (1) and (8), the level of very oblique wave amplitude necessary for equatorially mirroring electrons to reach the trapping island in phase space around exact Landau resonance (and thus eventually being affected by resonant interactions) is given approximately by

$$\frac{E_{w,\text{total}}}{cB_0} \geq \frac{\Omega_{ce}}{\Omega_{pe}}\frac{\gamma^2(q-1)}{4\,q^2}\Big/\sqrt{\frac{\Omega_{ce}}{\omega}-1} \tag{9}$$

The corresponding necessary wave amplitudes are plotted in Figure 2 for $\omega = 0.3\Omega_{ce}$ and $\Omega_{pe}/\Omega_{ce} = 6$ at $L \sim 6$. Such very oblique waves should have total magnetic amplitudes smaller than 500 pT to be considered as realistic [*Agapitov et al.*, 2014a; *Mourenas et al.*, 2015b]. Combined with equation (9), this leaves us only with waves close enough to their resonance cone angle, with $q < 1.1$. By virtue of conservation of the first adiabatic invariant, an electron scattered through Landau resonant interactions from $\alpha_0 \sim 90°$ down to $\sim 75°$ will see its energy increased by a mere 7%, which remains negligible.

It turns out that very oblique quasi-electrostatic chorus waves with realistic parallel electric field components $E_{w\parallel} \sim 5-50$ mV/m could impact all 30–300 keV electrons close to $\alpha_0 = 90°$ in regions of sufficiently high ratio $\Omega_{pe}/\Omega_{ce} > 5$ in the Earth's radiation belts, efficiently transporting them toward smaller pitch angles where nonlinear scattering by cyclotron resonance may take over and send them to even lower α_0 values. As a result, very oblique chorus waves considered alone can lead to the formation of butterfly pitch angle distributions with a relatively narrow trough around 90°. But they might also be one important component, together with parallel waves, in some fast electron dropouts in the outer radiation belt.

4. Combined Nonlinear Effects of Very Oblique and Parallel Chorus Waves on Electron Distributions

4.1. Parallel and Very Oblique Downgoing Chorus Waves

As shown in sections 2 and 3, Landau resonance with oblique lower band chorus waves is more favorable to impact electrons closer to $\alpha_0 = 90°$. Accordingly, let us consider a situation where intense, alternate bursts of parallel and very oblique lower band chorus waves are generated close to the equator [*Omura et al.*, 2009; *Shklyar and Matsumoto*, 2009; *Agapitov et al.*, 2014a; *Santolík et al.*, 2014; *Mourenas et al.*, 2015b]. When considering only such downgoing waves coming directly from their source region, electron trapping in Landau resonance results in pitch angle shifts $(\Delta\alpha_0)_{\text{trap}} < 0$ toward smaller pitch angles, while nonlinear Landau resonant scattering (i.e., phase bunching) leads to opposite $(\Delta\alpha_0)_{\text{NLscat}} > 0$ shifts corresponding to decreases in energy by virtue of conservation of the first adiabatic invariant [*Artemyev et al.*, 2014b]. For a fast and significant depletion of the electron distribution at high pitch angles to occur, two conditions must therefore be fulfilled: (1) nonlinear effects must be significant and (2) the time-integrated negative pitch angle shift due to Landau resonant trapping must be larger than the time-integrated positive shift produced by nonlinear Landau resonant scattering over the long run. The first condition provides lower bounds on the wave amplitude above which the so-called inhomogeneity ratio $S < 1$ at resonance, corresponding to the presence of closed trajectories in the phase plane [*Albert*, 2002; *Omura et al.*, 2009; *Artemyev et al.*, 2014b; *Mourenas et al.*, 2015b]. It is generally of the order of levels given by equation (9), but it depends on wave and plasma latitudinal profiles. The second condition is approximately equivalent to a relationship

$$(1 - \Pi_{\text{trap}})(\Delta\alpha_0)_{\text{NLscat}} < \Pi_{\text{trap}}(\Delta\alpha_0)_{\text{trap}} \tag{10}$$

where $\Pi_{\text{trap}} \leq 1$ denotes the probability of trapping (i.e., the portion of resonant particles which will get trapped during their first passage through the resonance). The corresponding analytical formulas for $(\Delta\alpha_0)_{\text{trap}}$, $(\Delta\alpha_0)_{\text{NLscat}}$, and Π_{trap} have been derived in previous works [*Artemyev et al.*, 2012, 2013b, 2014b, 2015c]. Thus, two lower bounds on the wave amplitude exist: a first lower bound for nonlinear effects to start developing and a second, higher, lower bound for trapping to dominate—which is therefore the only important one for us here.

We consider downgoing very oblique chorus waves with constant $q = 1.005$ or 1.03 generated over $\simeq 2°$ of magnetic latitude from the equator (i.e., their amplitude increases over that region) with $\omega/\Omega_{ce} = 0.35$ at the equator at $L \sim 5-6$ ($q = 1.005(1.03)$ corresponds to a refractive index $N = 200(100)$ for $\Omega_{pe}/\Omega_{ce} = 6$). Note, however, that the value of $p\cos\alpha_0$ from equation (1) corresponding to Landau resonance with very oblique waves of fixed q is independent of the ratio ω/Ω_{ce}: hence, the following conclusions should actually apply to other ratios in the same range $\sim 0.1-0.4$.

The lower bound on the wave amplitude for trapping to occur is relatively high: $B_{w,\text{total}}^{\min} \sim 100$ pT (or $E_{w\parallel}^{\min} \sim 25$ mV/m) in Figure 4 (see Appendix A for details on the numerical model). What is more surprising is the very narrow pitch angle range where Landau resonant trapping can occur. Depending on electron energy and wave obliquity (q parameter), it can only be encountered over a very small pitch angle interval of about 0.15° to 0.5° width, around a value of α_0 which varies between ~85° and ~89°. A naive interpretation might lead one to believe that trapping should be very inefficient and the corresponding downward pitch angle shifts negligible. However, the reverse happens to be true *over the long run* in a dynamical system where both nonlinear scattering (upward pitch angle shifts) and trapping (downward shifts) are possible, as in the case of repeated bounce motions of electrons along a given geomagnetic field line. Indeed, the results of test

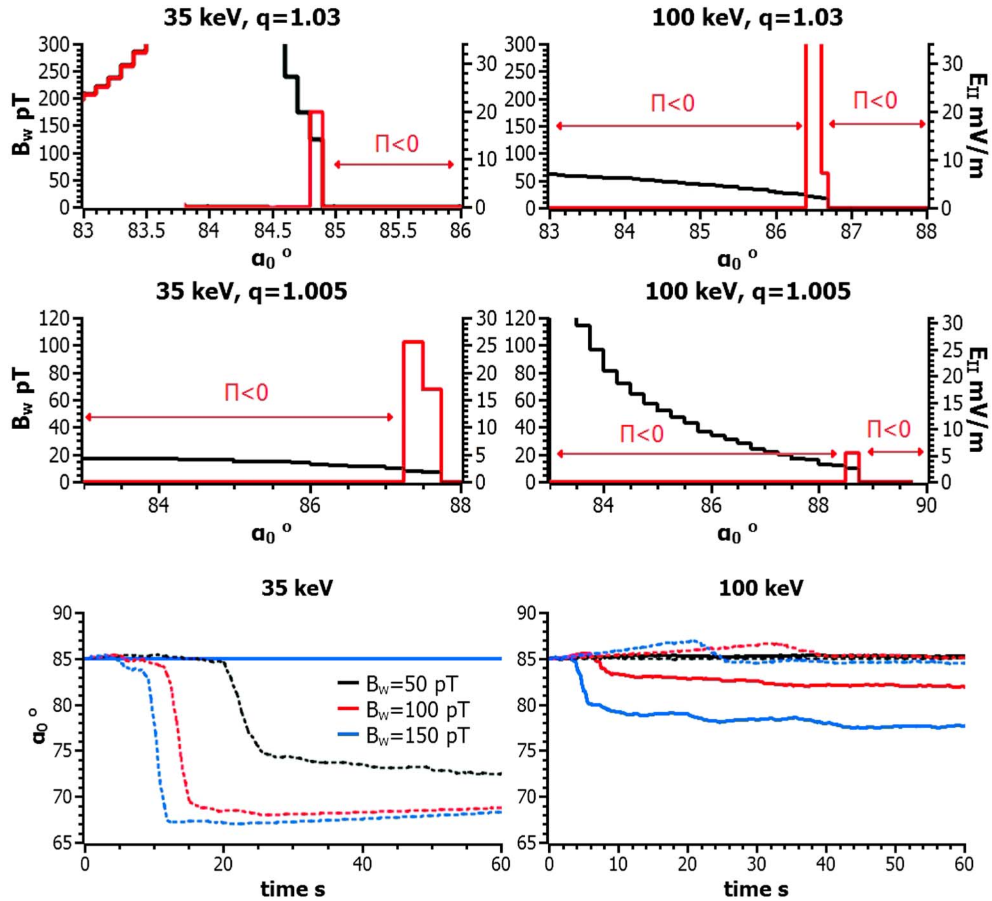

Figure 4. (top and middle rows) Lower bounds on the amplitude of downgoing very oblique chorus waves allowing nonlinear effects (in black) and trapping (in red) of electrons, as a function of α_0 for initial electron energy (left column) $E = 35$ keV and (right column) 100 keV. Other parameters are $q = 1.03$ (top row) and $q = 1.005$ (middle row), with $\omega/\Omega_{ce} = 0.35$, and $\Omega_{pe}/\Omega_{ce} = 6$. (bottom row) Average pitch angle advection obtained from 10^5 individual test particle simulations in the presence of both trapping and nonlinear scattering by the same waves as a function of time. We consider the same parameters as above, with $B_{w,\text{total}} \sim 50$, 100, and 150 pT for $q = 1.03$ (solid lines) and $q = 1.005$ (dashed lines).

particle simulations displayed in Figure 4 (bottom row) demonstrate that significant downward pitch angle shifts $\Delta\alpha_0 \sim -5°$ to $\sim -20°$ suddenly occur after 10–30 s due to Landau resonant trapping.

The scenario for these particular trapping events can be deciphered by examining one individual particle trajectory displayed in Figure 5. In this case, both parallel and very oblique (with $q = 1.03$) downgoing chorus waves are assumed to be present, with similar amplitude latitudinal profiles. We consider a given 100 keV electron, starting initially from a pitch angle domain located *below* the narrow range where Landau resonant trapping by the oblique wave is available. The considered electron experiences repeated nonlinear Landau resonant scatterings by this wave, progressively increasing its pitch angle, until it reaches the narrow α_0 domain of dominant trapping. There, a very efficient Landau resonant trapping in the oblique wavefield leads to a fast jump (over a fraction of the bounce period) to α_0 values much smaller than the initial value. It is the presence of Landau resonant nonlinear scattering by the oblique waves and the related upward pitch angle advection all over the α_0 range below the domain of allowed Landau resonant trapping that impels electrons to reach this very domain—where they get finally trapped and sent to much smaller α_0. The narrow Landau resonant trapping domain therefore represents a kind of *attractor* for particles initially located at lower pitch angles. Once they reach this attractor, particles ultimately end up being sent by one stronger trapping-related jump to the pitch angle range $\alpha_0 < 75°$. This whole process takes about ~50 s of a continuous presence of the waves for the electron trajectory displayed in Figure 5—but only 15–20 s on average (statistically) in Figure 4, i.e., probably a few minutes when considering a realistic sequence of isolated wave bursts.

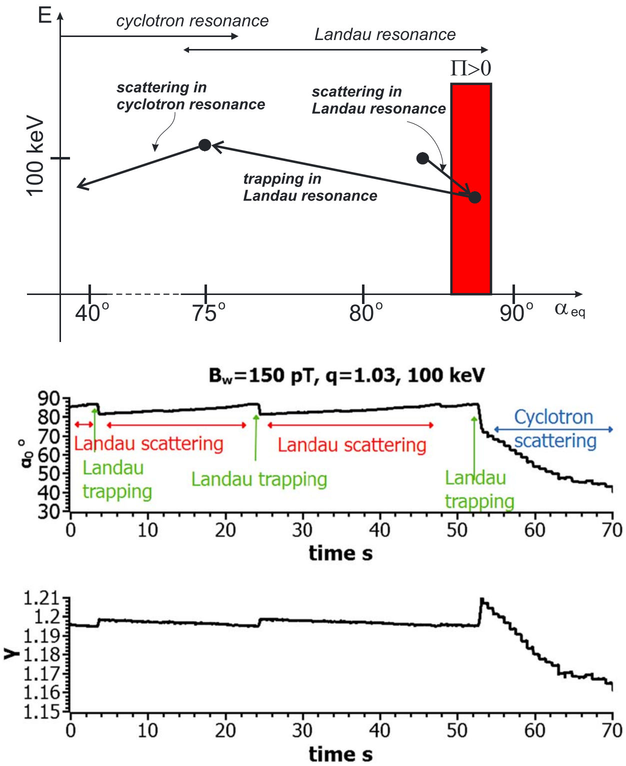

Figure 5. (top) Scheme of the successive nonlinear effects undergone by the electron. (middle) Individual test particle trajectory for $B_{w,\text{total}} = 150$ pT downgoing parallel chorus waves and very oblique chorus waves (with $q = 1.03$), for an initial electron energy $E = 100$ keV. Other parameters are $\omega/\Omega_{ce} = 0.35$, and $\Omega_{pe}/\Omega_{ce} = 6$. (bottom) Corresponding temporal variation of the electron γ.

Simulation results in Figure 4 (bottom row) show that such downward pitch angle jumps due to Landau resonant trapping by oblique waves can be 3–5 times larger for $E = 35$ keV than for 100 keV. However, it requires waves significantly closer to the resonance cone angle (i.e., with $q < 1.02$) than for higher energy electrons at $E \sim 100$ keV. If very oblique waves are present alone, the resulting pitch angle distributions may assume butterfly shapes with a narrow trough in the range $\alpha_0 \approx 75°–105°$.

But another realistic situation that can occur in the radiation belts is the nearly simultaneous presence (in time or MLT) on the same L shell of both parallel and very oblique lower band chorus waves generated near the equator in bursts of relatively high amplitudes. Figure 6 shows the results of test particle simulations concerning the average pitch angle advection of electrons by intense parallel chorus waves with $B_{w,\text{total}} = 50$ pT or 150 pT. Very strong and rapid downward pitch angle shifts are found due to cyclotron resonant nonlinear scattering [e.g., see *Albert*, 2002; *Artemyev et al.*, 2015c]—indeed much faster than for oblique waves of similar

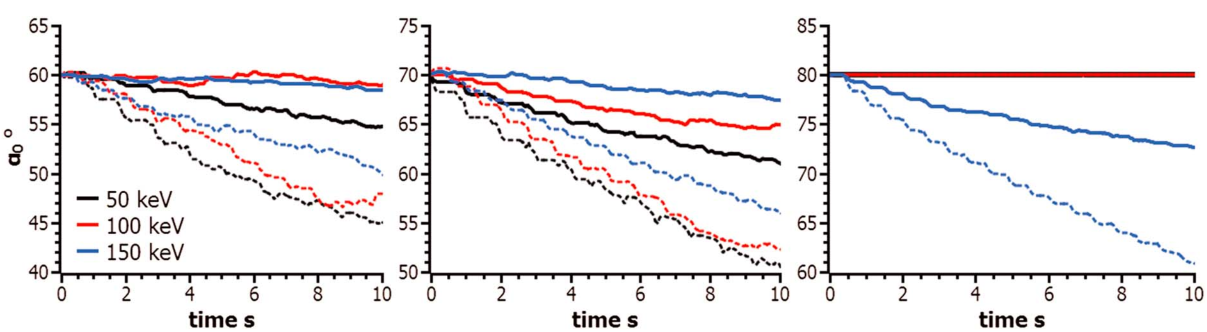

Figure 6. Electron pitch angle advection obtained from test-particle simulations in the presence of both trapping and nonlinear scattering by parallel whistler mode waves as a function of time, for $B_{w,\text{total}} \sim 50$ pT (solid curves) and 150 pT (dashed curves). Three different initial α_0 values are considered, as well as three different electron energies $E = 50$, 100, and 150 keV.

amplitudes (e.g., compare with Figure 4 for 150 pT and 100 keV) and sufficient to send the particles from $\alpha_0 \sim 70°$ down to $\alpha_0 \sim 50°$ in less than 10 s. In real events, moreover, magnetic field bursts of parallel waves are usually more intense than for oblique waves [*Santolík et al.*, 2014].

If parallel and very oblique wave bursts are generated in rapid succession over a sufficiently long time period (>3 min typically), then energetic electrons first transported via Landau resonant trapping by oblique waves to $\alpha_0 < 75°$ after about 2–3 min (see Figure 4) can later experience cyclotron resonance with parallel waves. Intense bursts of parallel waves can further nonlinearly scatter these electrons to $\alpha_0 \leq 40°$ in less than 1 min (see Figure 6 and the right side of Figure 5), simultaneously reducing their energy by about ≈20%. The proposed mechanism of double resonance (first Landau, then cyclotron) with a mixture of successive (in time or possibly in MLT) parallel and very oblique chorus waves could therefore account for the formation of butterfly pitch angle distributions of 30–150 keV electrons with a maximum at $\alpha_0 \leq 40°$. Moreover, electrons very close to $\alpha_0 = 90°$ are unaffected by the waves, and pitch angle transport in the whole range $\alpha_0 \sim 75°–90°$ is much weaker (i.e., slower) than for $\alpha_0 \sim 40°–75°$. In such a situation, the equilibrium electron distribution (obtained here after minutes of interactions) should present a relative maximum around $\alpha_0 = 90°$ as compared to the range $\alpha_0 \sim 40°–75°$(e.g., see section 2.3 in the work by *Mourenas et al.* [2015a] for a detailed discussion of a similar situation). Thus, the full pitch angle distribution should have a large maximum at $\alpha_0 < 40°$, a significant trough in the domain $\alpha_0 \sim 40°–75°$, and some weak secondary maximum around $\alpha_0 = 90°$.

If parallel chorus waves of average amplitudes ∼50 pT to 100 pT are also present at later MLT for about 30–60 min more, then the main (low pitch angle) part of these butterfly distributions of 30–150 keV electrons could ultimately get precipitated into the atmosphere in less than 1 h by quasi-linear diffusion [*Mourenas et al.*, 2012; *Artemyev et al.*, 2013a] or be transported to a lower energy channel—both processes possibly resulting in electron flux dropouts in given energy ranges. But a key requirement is that the double resonance mechanism of nonlinear transport from high to low equatorial pitch angles by oblique and parallel intense waves be efficient enough at earlier times. Otherwise, if earlier bursts of oblique and parallel waves are not intense or numerous enough, the main part of the electron population at $\alpha_0 > 50°–75°$ will instead remain in place and get accelerated via quasi-linear diffusion by the same ∼50–100 pT parallel chorus waves [*Horne and Thorne*, 2003; *Shprits et al.*, 2008b; *Thorne*, 2010; *Mourenas et al.*, 2014, 2015a].

The proposed mechanism of double resonance (first Landau, then cyclotron) with both parallel and very oblique intense chorus waves could therefore lead to the formation of butterfly pitch angle distributions and even ultimately develop into rapid dropouts of 30–150 keV electrons, provided that the wave amplitudes and obliquities lie in the appropriate ranges.

It is also worth emphasizing that for $\alpha_0 > 80°$ and $E > 150$ keV, the efficiency (i.e., probability and magnitude of individual downward pitch angle jumps) of Landau resonant trapping by very oblique waves strongly abates, while nonlinear scattering by parallel waves becomes less efficient for $\alpha_0 > 75°$ and $E > 200$ keV as trapping acceleration starts to prevail, increasing the pitch angles of electrons [*Artemyev et al.*, 2012, 2015c]. At very low energy $E < 30$ keV, conversely, oblique waves must lie extremely close to their resonance cone angle to trap high pitch angle electrons (e.g., see Figures 1 and 4). As a result, electron butterfly distributions occurring this way should be confined to the energy range $E \sim 30–150$ keV in general.

The proposed mechanism of butterfly distribution formation (possibly leading to dropouts) is mainly efficient when Landau resonance with oblique waves is available up to high pitch angles close to 90°, and the amplitude of oblique waves satisfies equation (9). This requires a relatively high ratio $\Omega_{pe}/\Omega_{ce} > 4$. For typical conditions outside the plasmasphere, it corresponds approximately to the region $L \geq 4$ [*Sheeley et al.*, 2001]. Finally, it should be emphasized that 5–20 keV electrons may be trapped and accelerated by the same intense oblique waves up to the range ∼100–200 keV, producing some increase of fluxes in this energy range [*Agapitov et al.*, 2015]. This acceleration process may compete with the loss mechanism proposed here, further confining potential dropouts to the range $E < 100–150$ keV.

An alternative scenario involving intense downgoing parallel chorus waves together with very oblique reflected waves of much smaller intensity has been examined in Appendix B. It could lead to similar effects, but it requires reflected wave amplitudes high enough (typically > 25 pT) over sufficiently long time periods (>10 min), a situation which is probably rarely encountered in the Earth's outer radiation belt.

4.2. Possible Observations in the Radiation Belts

We have searched the available data from the recent Van Allen Probes for some possible evidence of the proposed double resonance nonlinear mechanism discussed in section 3. Ideally, one would like to obtain wave measurements at magnetic latitudes $\lambda \simeq 2^\circ - 5^\circ$, where chorus waves generated near the equator are expected to attain their maximum amplitude [*Omura et al.*, 2009; *Shklyar and Matsumoto*, 2009; *Mourenas et al.*, 2015b], as well as particle measurements at $\lambda \leq 1.5^\circ$ allowing to see the whole distribution in equatorial pitch angle up to $\alpha_0 \geq 87^\circ$. However, these two contradictory requirements cannot be satisfied simultaneously. Thus, we have selected observations taking place around $\lambda \simeq 2^\circ - 5^\circ$, where measured electron pitch angles $\alpha = 90^\circ$ correspond to equatorial values $\alpha_0 \sim 80^\circ - 86^\circ$ (i.e., high enough to see a possible secondary maximum at $\alpha_0 \geq 80^\circ$, see section 3), while chorus waves should already have reached significant amplitudes there.

However, it is often difficult in practice to determine with sufficient accuracy all the needed plasma and magnetic field parameters from satellite measurements, especially during the somewhat disturbed periods at $L > 5$ that we are interested in. It is even harder to unambiguously assign a single cause (like our proposed mechanism) to an observed change in the electron distribution, because of the multitude of phenomena which are susceptible to occur simultaneously, or in close succession, in this region of space during disturbed periods. In particular, intense chorus emissions are likely to take place during periods of high $AE > 200-300$ nT following injections of 1–30 keV electron populations from the plasmasheet necessary for their generation [*Meredith et al.*, 2012; *Mourenas et al.*, 2015b]. Such populations may be convected and accelerated, leading to changes in the 30–100 keV distribution that mingle with (and possibly blur) the modifications locally induced by chorus waves. As concerns the nonlinear mechanism proposed here, we shall therefore provide one nice observation with adequate parameters, for which we can confirm the presence of the causing factors (the oblique and parallel chorus waves). Then, we shall assess whether the expected consequences on the particle distribution have been simultaneously measured or not—but without being able to definitely prove that the proposed mechanism was the sole (or even the main) determining cause.

The proposed double resonance mechanism for the formation of butterfly pitch angle distributions of 30–150 keV electrons may have been present during one particular event recorded on 13 February 2013 by the Van Allen Probes. On that day, data from Electric and Magnetic Fields Instrument Suite and Integrated Science (EMFISIS) on board the Van Allen Probe A [*Kletzing et al.*, 2013] reveal that from 1:50 UT to 3:00 UT, both parallel and very oblique lower band chorus waves coming from the equator were present at $L \sim 5.6$ around 2:00–2:30 MLT. Comparisons with different Tsyganenko models (T01, T01s, and T05) [see *Tsyganenko and Sitnov*, 2005] show that the geomagnetic field strength measured on board Van Allen Probe A was smaller than in the models, which give low magnetic latitudes around 4°–5°. This suggests that the spacecraft was probably even closer to the magnetic equator. Very oblique chorus waves propagating within less than 2°–3° from their resonance cone angle $\theta_r \sim 73^\circ - 75^\circ$ can be seen regularly from 1:00 UT to 3:00 UT in Figure 7 at frequencies $\omega \sim 0.3\Omega_{ce}$, with burst amplitudes $B_{w,\mathrm{total}} \approx 0.1$ nT. Intense bursts of parallel waves can also be distinguished at frequencies $\omega \sim 0.3-0.38\Omega_{ce}$, with maximum amplitudes $B_{w,\mathrm{total}} \approx 0.5$ nT. They show up as large peaks of wave magnetic field interspersed among quasi-electrostatic very oblique waves of smaller $B_{w,\mathrm{total}}$ but higher electric field $E_{w,\mathrm{total}}$. Sometimes both kinds of waves can be seen almost simultaneously at different frequencies (as near 2:32 UT), but, in general, parallel and very oblique waves are recorded alternatively in time.

During that whole 2 h period, magnetic disturbances remained small with $D_{\mathrm{st}} \simeq -4$ nT and $K_p \simeq 2$. However, an injection of 10–300 keV electrons took place near 1:40 UT, corresponding to a sudden (but moderate) increase of the AE index from 0 to 200–250 nT. Afterward, from 2 UT to 3 UT, AE remained nearly constant. Making use of radial diffusion rates D_{LL} derived by *Ozeke et al.* [2014] from ULF wave measurements, one finds a negligible radial diffusion $\Delta L < 0.1$ of electrons over $\Delta t = 60$ min for $K_p = 2$ and $L = 5.6$. However, one cannot guarantee that the evolution of electron fluxes was driven only locally on this L shell during this short period, because convection by large-scale electric fields is often more important than radial diffusion at $E < 100$ keV.

Over the same period from 1:50 UT to 3:00 UT during which alternate very oblique and parallel intense chorus waves were recorded by EMFISIS, simultaneous measurements from Magnetic Electron Ion Spectrometer (MagEIS) [*Blake et al.*, 2013] on the Van Allen Probe A show a first general increase of 35–150 keV electron fluxes until about 2 UT. Afterward, however, the 37 keV flux continued to climb slowly and 180–220 keV fluxes barely changed, while fluxes in the 56–110 keV range distinguished themselves by a pronounced drop

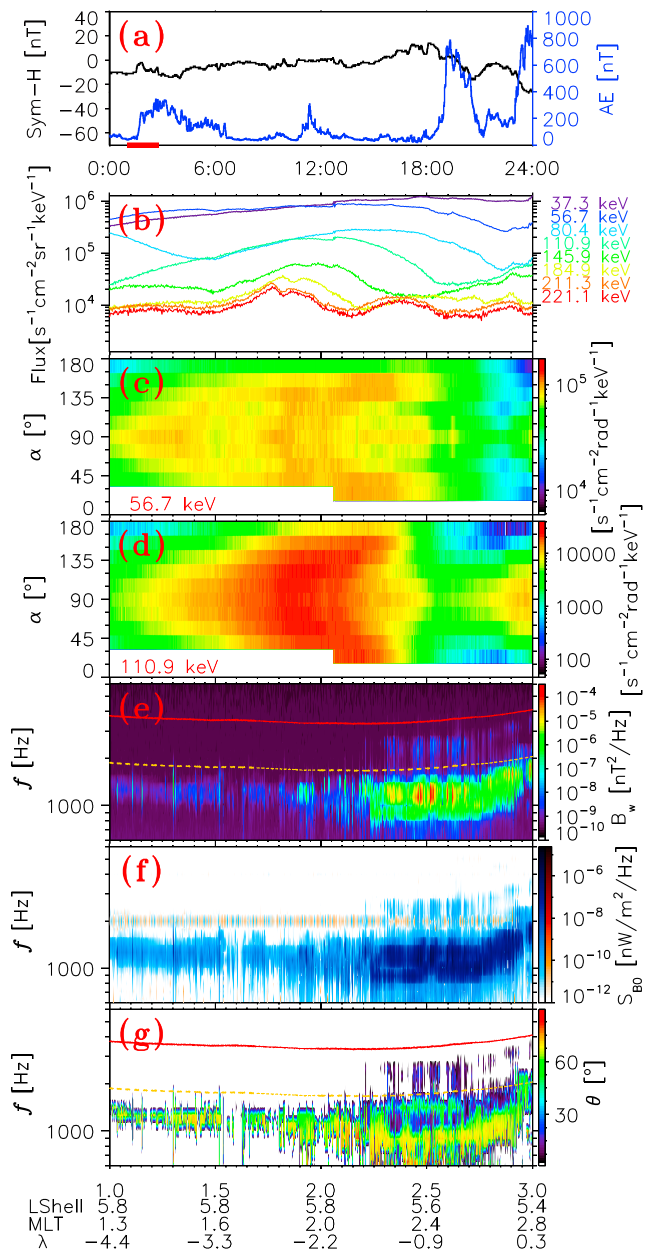

Figure 7. Observations of very oblique and parallel lower band chorus waves and simultaneous variations of electron fluxes, collected on board the Van Allen Probe A on 13 February 2013. (a) *AE* and *SYM-H* variations on that day, with a red mark showing the 2 h event from 1:00 UT to 3:00 UT considered below. (b) The temporal variation of total electron fluxes in various MagEIS energy channels: 37 keV, 56 keV, 80 keV, 110 keV, 146 keV, 185 keV, 211 keV, 221 keV (from top to bottom). (c) and (d) The evolution of the pitch angle distribution of 56 keV and 110 keV electrons from MagEIS during that 2 h period. (e) Magnetic field spectrograms (in nT^2/Hz) from EMFISIS (the local electron gyrofrequency f_{ce} and $0.5f_{ce}$ based on in situ magnetic field measurements are indicated by solid red and orange dashed lines, respectively). (f) The corresponding negative Poynting flux indicates that all the waves are propagating away from the equator. (g) The wave normal angle θ of the waves.

starting around 2:10–2:20 UT. Another key feature differentiates fluxes in the 56–110 keV range from lower and higher energy fluxes: the evolution of their pitch angle distribution. Pitch angle distributions at lower and higher energies both kept their usual shape $f(\alpha_0) \approx |\sin^{3/2}\alpha_0|$ throughout the period, while the distribution of 56–110 keV electrons changed to a characteristic butterfly shape, later keeping it for the next ~25 min to 50 min or so (see Figure 7). Moreover, a second, weaker maximum can be noticed near α_0 ~85°–88° in 56–110 keV electron distributions. Although a much weaker butterfly-like distribution can also be discerned at 56 keV from 1:35 UT to 2 UT, it could also be due to the presence of some weaker bursts of oblique chorus waves after 1 UT.

The rapid temporal decays of 56 keV and 80 keV electron fluxes between about 2:10 UT and 2:50 UT in Figure 7b are nearly identical and simultaneous. Moreover, these flux drops coincide with the period of observation of intense oblique and parallel chorus waves. Lastly, the 50–100 keV pitch angle distribution is simultaneously changed to a clear butterfly distribution with a weaker maximum near $\alpha_0 = 90°$—a distinctive *E* shape distribution—contrasting with distributions at lower and higher energies. Taken together, these three facts argue in favor of a presence of the proposed double-resonance nonlinear transport and loss mechanism. Simultaneous flux drops at different energies cannot be explained by a scenario of limited electron injections followed by azimuthal drift (due to the different drift velocities). Of course, an adequate combination of past injections with more recent ones, together with appropriate particle convections and drift, could also produce the observed flux drops as well as *E* shape pitch angle distributions. But this would require a very precise fine tuning of various phenomena, such that all these effects occur in the end exactly at the same time as the intense wave emissions. Although this alternative scenario cannot be ruled out, we find the coincidence of intense oblique and parallel chorus waves with all their expected consequences on the electron distribution (*E* shapes and drops in the predicted energy range) rather compelling. Both the characteristic *E* shape of the pitch angle distribution and its limited energy range (50–110 keV) agree well with numerical simulations (see Figures 4–6) performed for parallel and very oblique ($q \sim 1.03$) chorus waves of amplitudes roughly similar to the observed ones.

The rapid drop in 50–80 keV electron fluxes occurring around 2:30 UT could result from the cumulated and successive effects of (i) double resonance with intense very oblique and parallel chorus waves and (ii) quasi-linear scattering by the same parallel waves toward the loss cone. Indeed, the main drop near 2:30 UT seems to coincide with strong bursts of parallel waves lasting about 5–10 min. The cessation of the drop after 2:50 UT also corresponds to a global reduction of the obliquity of very oblique waves, as well as to an increase of the frequency of parallel waves which should reduce quasi-linear pitch angle scattering [*Mourenas et al.*, 2012]. The faster decrease of ∼110–150 keV fluxes, as well as their actual increase after about 2:30 UT, can be simply due to large natural oscillations seen in this energy range during the whole period from 1 UT to 3 UT. But the weaker flux drop at 56 keV than at 110 keV could also be due to an arrival of electrons from the higher neighboring energy channel comparatively more important at 56 keV as a result of cyclotron resonant electron transport toward lower pitch angles (see section 3). The 110–150 keV flux increase seen after 2:30 UT might also partly stem from an acceleration of initially ∼10–20 keV electrons by oblique and then parallel chorus waves, which should mainly result in an increase of fluxes in the range ≈ 120–200 keV (see corresponding observations and simulations in the works by *Agapitov et al.* [2015], *Albert* [2002], and *Artemyev et al.* [2015c]).

In the end, it must be acknowledged that an alternative sequence of other phenomena could also have caused the observed electron distribution changes, such as a combination of past and recent injections, together with convection, acceleration and drift. Some indications of dipolarization events near 23 UT on 12 February and near 2:10 UT on 13 February can be found in GOES 13 and Van Allen Probe B data, accompanied by injections of 30–50 keV electrons on the duskside. However, only a very small jump of 30–100 keV fluxes can be discerned near 2:05 UT on board the Van Allen Probe A at 2 MLT. Anyway, the event recorded on 13 February 2013 provides at least one conclusive evidence: it demonstrates that the parallel and very oblique intense chorus waves needed for the proposed double resonance mechanism of formation of 30–150 keV electron butterfly distributions (and potentially dropouts) can really exist together in the radiation belts over a time period of 60 min at $L \sim 5.5$, following particle injections from the plasma sheet (i.e., after $AE > 200$ nT intervals). In the future, it will be worthwhile to search in satellite data for other similar observations in order to provide more conclusive evidence concerning the efficiency of the proposed double resonance mechanism. Checking the occurrences of similar events would also help to assess the potential importance of this dual resonance mechanism in the dynamics of energetic electrons.

5. Conclusions

Accurately modeling and forecasting the wild variations of 30–150 keV electron fluxes recorded onboard satellites in the outer radiation belt may ultimately require to include almost all of the related phenomena. A number of physical processes may account for the formation of butterfly pitch angle distributions and losses of energetic electrons to the atmosphere or to the magnetopause boundary. In the present paper, we suggest that a mechanism of nonlinear electron transport by double resonance (Landau and cyclotron) with very oblique and parallel whistler mode chorus waves could also play a role at $L \sim 4$–6, especially during nonstorm times which are often less favorable to other mechanisms.

In particular, we have shown that both nonlinear Landau resonant interactions with oblique waves and cyclotron resonant interactions with parallel waves could be important: the first ones for quickly transporting from $\alpha_0 > 75°$ to lower α_0 nearly equatorially mirroring electrons which otherwise cannot be removed by parallel waves and the second ones to scatter them farther down toward the loss cone. This double-shot mechanism may produce pronounced butterfly pitch angle distributions of energetic 30–150 keV electrons in a few minutes in the presence of intense bursts of oblique and parallel waves. If parallel chorus waves remain intense enough afterward over at least 30 min, some dropouts of electron fluxes may even occur after the mainly low pitch angle electrons have been finally scattered into the loss cone.

Test-particle simulations of the corresponding trapping and nonlinear scattering processes, performed with realistic wave amplitudes and obliquities, appear to be consistent with some recent satellite observations. Typically, less than 2–3 min of nonlinear transport via Landau resonant interactions with intense bursts of very oblique waves can bring high pitch angle particles to the convenient pitch angle range where cyclotron resonant interactions with parallel waves can provide the final stroke necessary to send these electrons to the range $\alpha_0 \leq 40°$, simultaneously reducing their energy by roughly 20%. Subsequent quasi-linear diffusion by parallel chorus waves may precipitate these electrons into the atmosphere—or they could be shifted to sufficiently lower energy to make them disappear from their initial energy channel.

Useful estimates of the required amplitude levels of oblique waves have been derived for downgoing (i.e., coming from the equator) as well as reflected very oblique waves. When analyzing satellite data, such ranges of necessary amplitudes should be helpful to check on the basis of measured wave characteristics whether or not particular butterfly electron distributions or dropouts can be explained by the proposed double-shot mechanism. As discussed during the interpretation of one selected event provided here, however, many different phenomena can potentially affect the electron distribution at $L > 4$ during disturbed periods. Thus, it will probably be a hard task to unambiguously assign a single cause, like our proposed mechanism, to some observed changes in the electron distribution.

Although Landau and cyclotron resonance interactions with whistler mode waves are therefore good candidates for explaining the transport of electrons away from $\alpha_0 \simeq 90°$, one should also assess the role played by other mechanisms in the presence of whistler mode waves. Pitch angle oscillations can occur near 90° as a result of the effect of the Lorentz force term on the motion of nearly equatorially mirroring electrons (e.g., see equation (16) in the work by *Nunn and Omura* [2015), due to either the parallel electric field of oblique waves or the transverse magnetic field of parallel waves. But for $E < 100$ keV, maximum possible shifts are small ($\Delta\alpha_0 < 1°$ at $L \sim 6$) for either parallel waves with $B_{w,\mathrm{total}} < 1$ nT or oblique chorus at $\theta = \theta_r - 1°$ with $E_{w\|} < 100$ mV/m. Nevertheless, such oscillations occur even for an *initial* $\alpha_0 = 90°$. Thus, they might help to explain why these particular electrons can be scattered even during relatively quiet geomagnetic conditions.

Bounce resonance should also be considered. In the presence of parallel electrostatic wavefields interacting resonantly with the bounce motion of electrons, the corresponding oscillating force can violate the second adiabatic invariant J of the particles while simultaneously conserving the first invariant M [*Roberts and Schulz*, 1968; *Schulz and Lanzerotti*, 1974]. The magnitude of such a bounce resonant scattering of energetic electrons by oblique chorus waves has been evaluated in Appendix C. Significant effects occur mainly when Landau resonance is simultaneously available with very oblique waves propagating near their resonance cone angle. After 1 h of bounce resonant scattering, the total pitch angle shift could reach a significant level $\Delta\alpha_0 \approx -3.5°(-7°)$ for $E = 300(35)$ keV and $\langle E_{w\|}^2 \rangle^{1/2} \sim 10$ mV/m, demonstrating that this effect should not be overlooked.

It is interesting to note that the role of oblique whistler mode waves in trapping and scattering 30–150 keV electrons with large pitch angles can be played by any electrostatic wave with a strong parallel electric field and a low enough phase velocity. For example, the strong electrostatic noise (the so-called time domain structures, see review by *Mozer et al.* [2015]) recently found in the radiation belts on the nightside at $L \geq 5$ during disturbed periods with $AE > 300$ nT has usually high enough amplitude (10–100 mV/m) and sufficiently low parallel phase velocity to resonantly interact with ∼10–100 keV electrons at pitch angles >85° and trap them into Landau resonance [*Artemyev et al.*, 2014c]. The corresponding parallel electron acceleration rapidly decreases pitch angles and results in the formation of field-aligned particle distributions [*Mozer et al.*, 2016]. Thus, the combined effects of strong electrostatic noise (instead of oblique chorus) and parallel whistler mode waves might lead to the same kind of dropouts as described in the present study.

Finally, we have examined the possible effects of EMIC waves via Landau resonance or bounce resonance. However, Landau resonance with electrons of energy $E > 35$ keV and pitch angle $\alpha_0 \leq 89°$ would require very oblique EMIC waves with $\theta > 80°$, which have never (to our knowledge) been observed with significant amplitudes in the Earth's inner magnetosphere [e.g., see *Saikin et al.*, 2015]. For realistically low levels of their parallel electric field [*Engebretson et al.*, 2007], bounce resonance with these waves should moreover remain weaker than bounce resonance with realistic whistler mode waves.

Appendix A: Scheme of Test Particle Tracing

To plot Figures 4–6 and A1, we numerically integrate an ensemble of 10^5 trajectories of particles. We use gyroaveraged relativistic equations of motion [see *Artemyev et al.*, 2015c]. For the background geomagnetic field, a curvature-free model is employed [*Bell*, 1984], while plasma density is assumed to be constant along

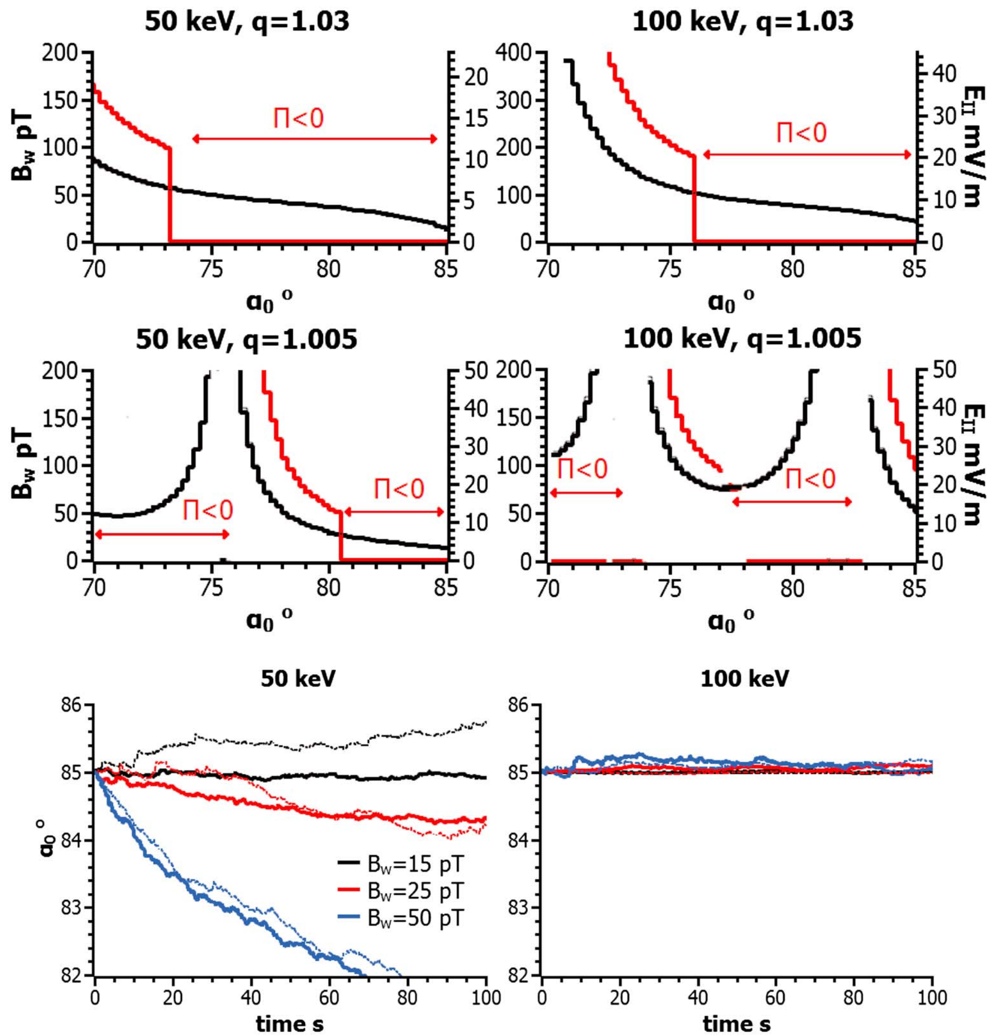

Figure A1. (top and middle rows) Lower bounds (in black) and upper bounds (in red) on the amplitude of very oblique reflected chorus waves potentially allowing a depletion of the electron flux in the presence of both trapping and nonlinear scattering, as a function of α_0, for initial electron energy $E = 50$ keV (left column) and 100 keV (right column). Other parameters are $q = 1.03$ (top row) and $q = 1.005$ (middle row), with $\omega/\Omega_{ce} = 0.35$, and $\Omega_{pe}/\Omega_{ce} = 6$. (bottom row) Electron pitch angle advection obtained from test-particle simulations in the presence of both trapping and nonlinear scattering by the same reflected waves as a function of time. We consider the same parameters as above, with $B_{w,\text{total}} \sim 15$, 25, and 50 pT for $q = 1.03$ (solid lines) and $q = 1.005$ (thin dashed lines).

magnetic field lines and provided by the model from *Sheeley et al.* [2001] at the magnetic equator. Wave electromagnetic fields are written through the vector and scalar potentials [see *Albert*, 1993; *Artemyev et al.*, 2015c] and averaged over electron gyrorotation. The relationships between components of wave electric and magnetic fields are taken in agreement with cold plasma dispersion theory [*Tao and Bortnik*, 2010]. According to the standard approach [*Shklyar and Matsumoto*, 2009], the wavefield is expanded over cyclotron resonances (i.e., wave electric and magnetic fields are each written as a sum of terms corresponding to resonance numbers n). We consider here only two resonances: $n = 0$ for oblique waves and $n = 1$ for parallel waves. Wave frequency is assumed to be constant (i.e., waves are monochromatic), while the wave number varies along magnetic field lines according to the dispersion relation for given θ [see *Artemyev et al.*, 2015c]. In the case of oblique waves, we keep $q = \cos\theta / \cos\theta_r$ fixed during wave propagation along magnetic field line. We also use a prescribed distribution of wave total amplitude (total magnetic field) along magnetic field line. This distribution corresponds to wave generation/damping within $\Delta\lambda \sim 2°$ of magnetic latitude around the equatorial plane, i.e., the wave amplitude is equal to zero at the equator and reaches its maximum value at $\lambda \sim 2°$, further staying constant at $\lambda > 2°$ (see details in *Artemyev et al.* [2015c]).

Appendix B: Combined Effects of Parallel and Reflected Very Oblique Chorus Waves

The amplitudes of very oblique waves needed to affect energetic electrons via Landau resonant nonlinear effects can be rather low (see Figure 2). Thus, one can consider a situation where intense bursts of parallel lower band chorus waves are generated close to the equator [*Omura et al.*, 2009; *Santolík et al.*, 2014], propagate toward higher latitudes, get refracted to higher wave normal angles, and damped, before being partially reflected when their frequency approaches the local lower hybrid frequency [*Kimura*, 1985; *Chen et al.*, 2013]. Reflected waves are often very oblique and, what is more important, they propagate toward the equator. In such a case, it is not unreasonable to assume that if overall Landau damping by suprathermal (100–500 eV) electrons is moderate enough to allow a partial reflection of the generated waves, but strong enough to fully damp them when they cross back the equator, then these waves will be reflected only once (at most) and come back toward the equator with only a small fraction ($\approx$ 10%) of their initial amplitude, as in observations [*Santolík et al.*, 2010].

Recent satellite data have shown that such a process of wave reflection (from higher L) and then strong damping close to the magnetic equator does exist, at least in the case of hiss waves in the plasmasphere [*Laakso et al.*, 2015]. The main consequence is that very oblique (reflected) waves propagating in a given direction will have an asymmetrical amplitude profile with respect to the equator, i.e., oblique waves come back from high latitudes toward the equator, but there are no oblique waves coming from the equator. This is really an important point, because such an asymmetrical latitudinal profile is much more propitious to obtain a significant advection toward smaller pitch angles through nonlinear scattering by reflected waves. Otherwise, waves propagating toward the equator from one side would scatter electrons to smaller pitch angles but, propagating away from the equator with the same amplitude on the other side, they would then scatter electrons to higher pitch angles, almost canceling the previous effect.

When considering reflected very oblique waves, however, the pitch angle shifts $(\Delta\alpha_0)_{\rm trap}$ due to trapping and $(\Delta\alpha_0)_{\rm NLscat}$ due to nonlinear scattering are oppositely directed toward higher and smaller α_0, respectively. When electrons coming from high latitudes get trapped by one wave, they remain for a while in Landau resonance at $v_\parallel = V_{{\rm ph}\parallel}$ with a wave phase velocity $V_{{\rm ph}\parallel}$ decreasing toward the equator like Ω_{ce}/Ω_{pe} (for a nearly constant q parameter), while $v_\parallel$ would otherwise increase (for an unperturbed α_0) due to conservation of the first adiabatic invariant. For the particle, it leads to a negative energy shift as well as a positive shift of α_0 due to the same adiabatic invariance. Conversely, nonlinear scattering via Landau resonance corresponds to a negative shift in α_0.

We consider reflected waves with an amplitude profile going down to zero below $\sim$2° of magnetic latitude. For a rapid depletion of the electron distribution at high pitch angles to occur, two conditions must be fulfilled: (1) nonlinear effects must be significant and (2) the time-integrated pitch angle shift due to nonlinear scattering must be larger than the time-integrated shift produced by trapping. The first condition provides lower bounds on the wave amplitude $B_w^{\rm min}$ (or equivalently $E_{w\parallel}^{\rm min}$). The second condition is approximately equivalent to a relationship opposite to (10). This new condition now imposes upper bounds $B_w^{\rm max}$ (or $E_{w\parallel}^{\rm max}$) on the amplitude of oblique lower band chorus waves able to produce a depletion of the electron flux at high α_0. These lower and upper bounds at $L \sim 5$–6 are plotted (in black and red, respectively) in Figure A1 as a function of electron pitch angle and energy for $q = 1.005(1.03)$ with $\omega/\Omega_{ce} \sim 0.35$ and $\Omega_{pe}/\Omega_{ce} = 6$ at the equator. For α_0 ~75°–85°, trapping often does not exist at all for $E \leq 100$ keV (corresponding here to $\Pi_{trap} \leq 0$). In the considered situation for $E = 50$–100 keV and over a wide region of parameter space (q, B_w), only lower bounds on the wave amplitude are present. At high enough amplitude, nonlinear scattering due to Landau resonance should prevail and strongly reduce the pitch angle of electrons. The required amplitudes of very oblique waves turn out to be moderate enough and therefore realistic, typically $B_{w,\text{total}} > 25(50)$ pT and $E_{w\parallel} > 4(8)$ mV/m for $E = 50(100)$ keV.

The average pitch angle advection of electrons obtained from 10^5 individual test particle runs is displayed in of Figure A1 (bottom row). Important downward pitch angle shifts of nearly equatorially mirroring particles occur due to their nonlinear Landau resonant interactions with reflected waves. The advection rate reaches $\partial\alpha_0/\partial t \approx -10^{-2}(-4 \cdot 10^{-2})$°/s for 50 keV electrons and $B_{w,\text{total}} = 25(50)$ pT. However, there is no such effect for 100 keV electrons. Butterfly pitch angle distributions can therefore be produced only at low energy $E <$ 100 keV (at least for $B_{w,\text{total}} \leq 50$ pT).

After a time long enough (>10 min), such a nonlinear transport can produce a time-integrated shift $\Delta\alpha_0 \approx -10°$ large enough to send particles from $\alpha_0 \simeq 85°$ toward smaller values (≤75°) where cyclotron resonance with parallel waves becomes available (see Figure 1). If intense parallel waves are indeed present, then these electrons can be very quickly nonlinearly transported toward $\alpha_0 \leq 40°$ while simultaneously loosing energy to parallel waves [*Albert*, 2002]. It has been shown in Figure 6 that downward pitch angle shifts due to cyclotron resonant nonlinear scattering are faster than for oblique waves of similar amplitudes (compare with Figure A1 for 50 pT and 50 keV). The pitch angle distribution of 30–100 keV electrons should then assume a butterfly shape, with a large maximum at $\alpha_0 \leq 40°$ and a weaker maximum near $\alpha_0 = 90°$.

The above mechanism of double resonance with both parallel chorus waves coming from their equatorial source region and (much less intense) oblique reflected waves could therefore produce important downward pitch angle advections, which might ultimately lead to rapid dropouts of 30–100 keV electrons when intense parallel waves remain present for a time long enough at later MLT. However, the most stringent requirement is the initial presence of very oblique reflected waves of high enough amplitude during a time period exceeding 10 min.

Appendix C: Pitch Angle Diffusion by Bounce Resonance With Oblique Whistler Mode Waves

In a static dipolar geomagnetic field, the oscillatory parallel force can be represented as a sum of Fourier components. Its average effect on particle pitch angles such that $\cos\alpha_0 \ll 1$ can be calculated for time periods much longer than one bounce period $\tau_b = 2\pi/\Omega_b$, with $\Omega_b = 2\pi p/(4mT(\alpha_0)LR_E)$ the bounce frequency, R_E the Earth's radius, and $T(\alpha_0) \simeq 3/4$ for $\alpha > 65°$ [*Schulz and Lanzerotti*, 1974]. In such a case, it has been shown [*Roberts and Schulz*, 1968; *Schulz and Lanzerotti*, 1974] that the evolution of the pitch angle distribution $F(\alpha_0, M, L)$ of electrons with $\alpha_0 > 65°$ should obey a Fokker-Planck diffusion equation of the form

$$\frac{\partial F}{\partial t} = \frac{y^3}{xT(y)}\frac{\partial}{\partial x}\left(\frac{xT(y)D_{xx}}{y^3}\frac{\partial F}{\partial x}\right)_{M,L} \tag{C1}$$

at constant M and L, with $x = \cos\alpha_0$, $y = \sin\alpha_0$, and a quasi-linear bounce-averaged diffusion (in x) coefficient D_{xx} given by

$$D_{xx} \simeq \frac{L^3 y^6 e^2}{m_0 M B_0}\sum_{n=1}^{+\infty}\frac{n^2 J_n^2(z_n)}{2z_n^2}\left(J_0(\eta) - \frac{B_{w,x}v_\perp}{cE_{w\parallel}}J_1(\eta)\right)^2\frac{\langle E_{w\parallel}^2\rangle}{\Delta f} \tag{C2}$$

where we consider bounce-resonant whistler mode waves with frequencies $\omega = n\Omega_b$, $k_\parallel c/\omega \simeq \Omega_{pe}\cos\theta/(\omega\epsilon)$, parallel electric field spectral density $\langle E_{w\parallel}^2(f)\rangle/\Delta f$ (with f and Δf the wave frequency and bandwidth, respectively), the argument of Bessel functions J_n is either $z_n = npxk_\parallel/(m\omega)$ or $\eta = k_\perp p_\perp/(m_0\Omega_{ce})$. The multiplication factor containing J_0 and J_1 terms arises after averaging the wavefield over gyromotion. Absent in the original formulation [*Schulz and Lanzerotti*, 1974], it becomes $\ll 1$ when dealing with very oblique waves such that $\eta > 1$.

Bounce resonant waves correspond to $n = \omega/\Omega_b \simeq 3f\gamma m_0 LR_E/p$, giving $n \approx 2500$ to 600 as L increases from 3 to 6.6 for $\omega/\Omega_{ce} \sim 0.3$ and $E = 50 - 300$ keV. Then, Bessel functions $J_n^2(z_n)$ in equation (C2) remain large only near $z_n = n$ [*Abramowitz and Stegun*, 1972]. Thus, significant bounce resonant scattering occurs mainly for $z_n \sim n$, i.e., in the domain wherein Landau resonance is available. In this case, equation (1) shows that the minimum pitch angle where bounce resonance can efficiently shift electrons varies like $\alpha_{0L} \propto \arccos(\gamma m_0 c/p)$, decreasing with electron energy E. Let us consider very oblique waves at $\theta \simeq \theta_r - X°$ with large parallel electric fields and $\omega/\Omega_{ce} \sim 0.3$ at $L \sim 5.5$, the maximum pitch angle $\alpha_{0,\max}$ where this scattering can affect particles is $\alpha_{0,\max} \sim 90° - 3(6)°/\sqrt{X}$ for $E = 300(35)$ keV. Keeping only the dominant contributions to D_{xx} for $z_n \sim n$, one finds $\eta \sim \gamma \tan\alpha_0 \sin\theta > 1$. Taking $B_{w,x} \simeq B_{w,\text{total}}/(\sqrt{2}\tan\theta)$, one gets $K(\alpha_0, \omega, \theta) = B_{w,x} v_\perp/(cE_{w\|}) \simeq \tan\alpha_0/[(1+\epsilon^{-2})\sin^2\theta]$. Asymptotic expressions of the Bessel functions for $\eta \gg 1$ give $J_m(\eta) \sim \sqrt{2/\pi\eta}\cos(\eta - m\pi/2 - \pi/4)$ [*Abramowitz and Stegun*, 1972], yielding

$$D_{xx}[\text{s}^{-1}] \sim \frac{2.2\,10^6 \sin^3\alpha_0 \cos\alpha_0}{(p/m_0c)\sin\theta\gamma^2 fL} \frac{\langle E_{w\|}^2(f)\rangle}{\Delta f} \times \left(\cos(\eta - \pi/4) - K(\alpha_0, \omega, \theta)\sin(\eta - \pi/4)\right)^2 \qquad \text{(C3)}$$

with f and Δf in Hz and $\langle E_{w\|}^2(f)\rangle$ in V^2/m^2. For $\alpha_0 \sim 85°$ and $\omega/\Omega_{ce} \sim 0.3$ at $L = 5.5$, $\theta = \theta_r$- 1°, and $\Delta f/f \sim 0.3$, one gets $D_{xx} \approx \langle E_{w\|}^2(f)\rangle/100 \approx 4 \cdot 10^{-7}(2 \cdot 10^{-6})$ s^{-1} for $E = 300(35)$ keV and a realistic level $\langle E_{w\|}^2\rangle^{1/2} \sim 0.01$ V/m. Since this quasi-linear bounce resonant scattering process is akin to a random walk in pitch angle space, one has approximately $(\Delta\cos\alpha_0)(t) \sim \sqrt{2D_{xx}t}$, giving $\Delta\cos\alpha_0 \sim 0.04(0.09)$ for $E = 300(35)$ keV after 60 min, corresponding to pitch angle shifts $\Delta\alpha_0 \approx -3.5°(-7°)$. When $\theta > \theta_g$, equations (5) and (C3) imply that D_{xx} is proportional to $\langle E_{w,\text{total}}^2\rangle(f/\Delta f)/(p\gamma L\Omega_{ce}^2)$. For similar wave electric power and ratio $\Delta f/f$, D_{xx} should be much larger at higher $L \sim 4.5-7$ than at lower L for most whister mode waves, with the possible exception of narrowband VLF waves at 12–25 kHz from powerful transmitters at $L = 1.3-2.2$ [e.g., see *Agapitov et al.*, 2014b].

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References

Abramowitz, M., and I. A. Stegun (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, John Wiley, New York.

Agapitov, O., A. Artemyev, V. Krasnoselskikh, Y. V. Khotyaintsev, D. Mourenas, H. Breuillard, M. Balikhin, and G. Rolland (2013), Statistics of whistler-mode waves in the outer radiation belt: Cluster STAFF-SA measurements, *J. Geophys. Res. Space Physics*, *118*, 3407–3420, doi:10.1002/jgra.50312.

Agapitov, O., A. Artemyev, D. Mourenas, V. Krasnoselskikh, J. Bonnell, O. Le Contel, C. M. Cully, and V. Angelopoulos (2014a), The quasi-electrostatic mode of chorus waves and electron nonlinear acceleration, *J. Geophys. Res. Space Physics*, *119*, 1606–1626, doi:10.1002/2013JA019223.

Agapitov, O., A. Artemyev, D. Mourenas, Y. Kasahara, and V. Krasnoselskikh (2014b), Inner belt and slot region electron lifetimes and energization rates based on AKEBONO statistics of whistler waves, *J. Geophys. Res. Space Physics*, *119*, 2876–2893, doi:10.1002/2014JA019886.

Agapitov, O., A. Artemyev, D. Mourenas, F. S. Mozer, and V. Krasnoselskikh (2015), Nonlinear local parallel acceleration of electrons through Landau trapping by oblique whistler-mode waves in the outer radiation belt, *Geophys. Res. Lett.*, *42*, 10,140–10,149, doi:10.1002/2015GL066887.

Albert, J. M. (1993), Cyclotron resonance in an inhomogeneous magnetic field, *Phys. Fluids B*, *5*, 2744–2750, doi:10.1063/1.860715.

Albert, J. M. (2002), Nonlinear interaction of outer zone electrons with VLF waves, *Geophys. Res. Lett.*, *29*(8), 1275, doi:10.1029/2001GL013941.

Albert, J. (2014), Radial diffusion simulations of the 20 September 2007 radiation belt dropout, *Ann. Geophys.*, *32*, 925–934, doi:10.5194/angeo-32-925-2014.

Artemyev, A., V. Krasnoselskikh, O. Agapitov, D. Mourenas, and G. Rolland (2012), Non-diffusive resonant acceleration of electrons in the radiation belts, *Phys. Plasmas*, *19*, 122901, doi:10.1063/1.4769726.

Artemyev, A., D. Mourenas, O. Agapitov, and V. Krasnoselskikh (2013a), Parametric validations of analytical lifetime estimates for radiation belt electron diffusion by whistler waves, *Ann. Geophys.*, *31*, 599–624, doi:10.5194/angeo-31-599-2013.

Artemyev, A. V., A. A. Vasiliev, D. Mourenas, O. V. Agapitov, and V. V. Krasnoselskikh (2013b), Nonlinear electron acceleration by oblique whistler waves: Landau resonance vs. cyclotron resonance, *Phys. Plasmas*, *20*(12), 122901, doi:10.1063/1.4836595.

Artemyev, A. V., A. A. Vasiliev, D. Mourenas, O. Agapitov, V. Krasnoselskikh, D. Boscher, and G. Rolland (2014a), Fast transport of resonant electrons in phase space due to nonlinear trapping by whistler waves, *Geophys. Res. Lett.*, *41*, 5727–5733, doi:10.1002/2014GL061380.

Artemyev, A. V., A. A. Vasiliev, D. Mourenas, O. V. Agapitov, and V. V. Krasnoselskikh (2014b), Electron scattering and nonlinear trapping by oblique whistler waves: The critical wave intensity for nonlinear effects, *Phys. Plasmas*, *21*(10), 102903, doi:10.1063/1.4897945.

Artemyev, A. V., O. Agapitov, F. Mozer, and V. Krasnoselskikh (2014c), Thermal electron acceleration by localized bursts of electric field in the radiation belts, *Geophys. Res. Lett.*, *41*, 5734–5739, doi:10.1002/2014GL061248.

Artemyev, A., O. Agapitov, D. Mourenas, V. Krasnoselskikh, and F. Mozer (2015a), Wave energy budget analysis in the Earth's radiation belts uncovers a missing energy, *Nat. Commun.*, *6*, 8143, doi:10.1038/ncomms8143.

Artemyev, A. V., D. Mourenas, O. V. Agapitov, and V. V. Krasnoselskikh (2015b), Relativistic electron scattering by magnetosonic waves: Effects of discrete wave emission and high wave amplitudes, *Phys. Plasmas*, *19*, 062901, doi:10.1063/1.4922061.

Artemyev, A. V., A. A. Vasiliev, D. Mourenas, A. I. Neishtadt, O. V. Agapitov, and V. V. Krasnoselskikh (2015c), Probability of relativistic electron trapping by parallel and oblique whistler-mode waves in Earth radiation belts, *Phys. Plasmas*, *22*, 112903, doi:10.1063/1.4935842.

Bell, T. F. (1984), The nonlinear gyroresonance interaction between energetic electrons and coherent VLF waves propagating at an arbitrary angle with respect to the earth's magnetic field, *J. Geophys. Res.*, *89*, 905–918, doi:10.1029/JA089iA02p00905.

Blake, J. B., et al. (2013), The Magnetic Electron Ion Spectrometer (MagEIS) instruments aboard the Radiation Belt Storm Probes (RBSP) spacecraft, *Space Sci. Rev.*, *179*, 383–421, doi:10.1007/s11214-013-9991-8.

Bortnik, J., and R. M. Thorne (2010), Transit time scattering of energetic electrons due to equatorially confined magnetosonic waves, *J. Geophys. Res.*, *115*, A07213, doi:10.1029/2010JA015283.

Chen, L., R. M. Thorne, W. Li, and J. Bortnik (2013), Modeling the wave normal distribution of chorus waves, *J. Geophys. Res. Space Physics*, *118*, 1074–1088, doi:10.1029/2012JA018343.

Chen, L. J., A. Maldonado, J. Bortnik, R. M. Thorne, J. Li, L. Dai, and X. Zhan (2015), Nonlinear bounce resonances between magnetosonic waves and equatorially mirroring electrons, *J. Geophys. Res. Space Physics*, *120*, 6514–6527, doi:10.1002/2015JA021174.

Elkington, S. R., M. K. Hudson, and A. A. Chan (2003), Resonant acceleration and diffusion of outer zone electrons in an asymmetric geomagnetic field, *J. Geophys. Res.*, *108*(A3), 1116, doi:10.1029/2001JA009202.

Engebretson, M. J., A. Keiling, K.-H. Fornacon, C. A. Cattell, J. R. Johnson, J. L. Posch, S. R. Quick, K.-H. Glassmeier, G. K. Parks, and H. Rème (2007), Cluster observations of Pc 1–2 waves and associated ion distributions during the October and November 2003 magnetic storms, *Planet. Space Sci.*, *55*, 829–848, doi:10.1016/j.pss.2006.03.015.

Gannon, J. L., X. Li, and D. Heynderickx (2007), Pitch angle distribution analysis of radiation belt electrons based on combined release and radiation effects satellite medium electrons a data, *J. Geophys. Res.*, *112*, A05212, doi:10.1029/2005JA011565.

Gao, X., W. Li, J. Bortnik, R. M. Thorne, Q. Lu, Q. Ma, X. Tao, and S. Wang (2015), The effect of different solar wind parameters upon significant relativistic electron flux dropouts in the magnetosphere, *J. Geophys. Res. Space Physics*, *120*, 4324–4337, doi:10.1002/2015JA021182.

Gu, X., Z. Zhao, B. Ni, Y. Shprits, and C. Zhou (2011), Statistical analysis of pitch angle distribution of radiation belt energetic electrons near the geostationary orbit: CRRES observations, *J. Geophys. Res.*, *116*, A01208, doi:10.1029/2010JA016052.

Horne, R. B., and R. M. Thorne (2003), Relativistic electron acceleration and precipitation during resonant interactions with whistler-mode chorus, *Geophys. Res. Lett.*, *30*(10), doi:10.1029/2003GL016973.

Horne, R. B., R. M. Thorne, S. A. Glauert, J. M. Albert, N. P. Meredith, and R. R. Anderson (2005), Timescale for radiation belt electron acceleration by whistler mode chorus waves, *J. Geophys. Res.*, *110*, A03225, doi:10.1029/2004JA010811.

Horne, R. B., S. A. Glauert, N. P. Meredith, D. Boscher, V. Maget, D. Heynderickx, and D. Pitchford (2013), Space weather impacts on satellites and forecasting the Earth's electron radiation belts with SPACECAST, *Space Weather*, *11*, 169–186, doi:10.1002/swe.20023.

Hwang, J., E.-J. Choi, J.-S. Park, M.-C. Fok, D.-Y. Lee, K.-C. Kim, D.-K. Shin, M. E. Usanova, and G. D. Reeves (2015), Comprehensive analysis of the flux dropout during 7–8 November 2008 storm using multisatellite observations and RBE model, *J. Geophys. Res. Space Physics*, *120*, 4298–4312, doi:10.1002/2015JA021085.

Jaynes, A. N., et al. (2015), Source and seed populations for relativistic electrons: Their roles in radiation belt changes, *J. Geophys. Res. Space Physics*, *120*, 7240–7254, doi:10.1002/2015JA021234.

Karpman, V. I. (1974), Nonlinear effects in the ELF waves propagating along the magnetic field in the magnetosphere, *Space Sci. Rev.*, *16*, 361–388, doi:10.1002/2015JA021234.

Kersten, T., R. B. Horne, S. A. Glauert, N. P. Meredith, B. J. Fraser, and R. S. Grew (2014), Electron losses from the radiation belts caused by EMIC waves, *J. Geophys. Res. Space Physics*, *119*, 8820–8837, doi:10.1002/2014JA020366.

Kim, K. C., D.-Y. Lee, H.-J. Kim, E. S. Lee, and C. R. Choi (2010), Numerical estimates of drift loss and Dst effect for outer radiation belt relativistic electrons with arbitrary pitch angle, *J. Geophys. Res.*, *115*, A03208, doi:10.1029/2009JA014523.

Kimura, I. (1985), Whistler mode propagation in the earth and planetary magnetospheres and ray tracing techniques, *Space Sci. Rev.*, *42*, 449–466, doi:10.1007/BF002149983.

Kletzing, C. A., et al. (2013), The Electric and Magnetic Field Instrument Suite and Integrated Science (EMFISIS) on RBSP, *Space Sci. Rev.*, *179*, 127–181, doi:10.1007/s11214-013-9993-6.

Laakso, H., O. Santolik, R. Horne, I. Kolmasová, P. Escoubet, A. Masson, and M. Taylor (2015), Identifying the source region of plasmaspheric hiss, *Geophys. Res. Lett.*, *42*, 3141–3149, doi:10.1002/2015GL063755.

Li, W., J. Bortnik, R. M. Thorne, C. M. Cully, L. Chen, V. Angelopoulos, Y. Nishimura, J. B. Tao, J. W. Bonnell, and O. LeContel (2013), Characteristics of the Poynting flux and wave normal vectors of whistler-mode waves observed on THEMIS, *J. Geophys. Res. Space Physics*, *118*, 1461–1471, doi:10.1002/jgra.50176.

Li, W., et al. (2014), Evidence of stronger pitch angle scattering loss caused by oblique whistler-mode waves as compared with quasi-parallel waves, *Geophys. Res. Lett.*, *41*, 6063–6070, doi:10.1002/2014GL061260.

Li, X., X. Tao, Q. Lu, and L. Dai (2015), Bounce resonance diffusion coefficients for spatially confined waves, *Geophys. Res. Lett.*, *42*, 9591–9599, doi:10.1002/2015GL066324.

Meredith, N. P., R. B. Horne, A. Sicard-Piet, D. Boscher, K. H. Yearby, W. Li, and R. M. Thorne (2012), Global model of lower band and upper band chorus from multiple satellite observations, *J. Geophys. Res.*, *117*, A10225, doi:10.1029/2012JA017978.

Millan, R. M., and R. M. Thorne (2007), Review of radiation belt relativistic electron losses, *J. Atmos. Sol. Terr. Phys.*, *69*, 362–377, doi:10.1016/j.jastp.2006.06.019.

Morley, S. K., R. H. W. Friedel, E. L. Spanswick, G. D. Reeves, J. T. Steinberg, J. Koller, T. Cayton, and E. Noveroske (2010), Dropouts of the outer electron radiation belt in response to solar wind stream interfaces: Global positioning system observations, *Proc. R. Soc. A*, *466*, 3329–3350, doi:10.1098/rspa.2010.0078.

Mourenas, D., A. V. Artemyev, J.-F. Ripoll, O. V. Agapitov, and V. V. Krasnoselskikh (2012), Timescales for electron quasi-linear diffusion by parallel and oblique lower-band Chorus waves, *J. Geophys. Res.*, *117*, A06234, doi:10.1029/2012JA017717.

Mourenas, D., A. V. Artemyev, O. V. Agapitov, and V. V. Krasnoselskikh (2013), Analytical estimates of electron quasi-linear diffusion by fast magnetosonic waves, *J. Geophys. Res. Space Physics*, *118*, 3096–3112, doi:10.1002/jgra.50349.

Mourenas, D., A. Artemyev, O. Agapitov, and V. Krasnoselskikh (2014), Consequences of geomagnetic activity on energization and loss of radiation belt electrons by oblique chorus waves, *J. Geophys. Res. Space Physics*, *119*, 2775–2796, doi:10.1002/2013JA019674.

Mourenas, D., A. Artemyev, O. Agapitov, V. Krasnoselskikh, and W. Li (2015a), Approximate analytical solutions for the trapped electron distribution due to quasi-linear diffusion by whistler-mode waves, *J. Geophys. Res. Space Physics*, *119*, 9962–9977, doi:10.1002/2014JA020443.

Mourenas, D., A. Artemyev, O. Agapitov, V. Krasnoselskikh, and F. S. Mozer (2015b), Very oblique whistler generation by low-energy electron streams, *J. Geophys. Res. Space Physics*, *120*, 3665–3683, doi:10.1002/2015JA021135.

Mozer, F. S., O. Agapitov, A. Artemyev, J. F. Drake, V. Krasnoselskikh, S. Lejosne, and I. Vasko (2015), Time domain structures: What and where they are, what they do, and how they are made, *Geophys. Res. Lett.*, *42*, 3627–3638, doi:10.1002/2015GL063946.

Mozer, F. S., A. Artemyev, O. Agapitov, D. Mourenas, and I. Vasko (2016), Near-relativistic electron acceleration by Landau trapping in time domain structures, *Geophys. Res. Lett.*, *43*, 508–514, doi:10.1002/2015GL067316.

Mulligan-Skov, T., J. F. Fennell, J. L. Roeder, J. B. Blake, and S. G. Claudepierre (2015), Internal charging hazards in near-Earth space during solar cycle 24 maximum: Van Allen Probes measurements, *IEEE Trans. Plasma Sci.*, *43*(9), 3070–3074, doi:10.1109/TPS.2015.2468214.

Nemec, F., O. Santolík, Z. Hrbácková, and N. Cornilleau-Wehrlin (2015), Intensities and spatiotemporal variability of equatorial noise emissions observed by the Cluster spacecraft, *J. Geophys. Res. Space Physics*, *120*, 1620–1632, doi:10.1002/2014JA020814.

Nunn, D., and Y. Omura (2015), A computational and theoretical investigation of nonlinear wave-particle interactions in oblique whistlers, *J. Geophys. Res. Space Physics*, *120*, 2890–2911, doi:10.1002/2014JA020898.

Omura, Y., M. Hikishima, Y. Katoh, D. Summers, and S. Yagitani (2009), Nonlinear mechanisms of lower-band and upper-band VLF chorus emissions in the magnetosphere, *J. Geophys. Res.*, *114*, A07217, doi:10.1029/2009JA014206.

Ozeke, L. G., I. R. Mann, K. R. Murphy, I. Jonathan Rae, and D. K. Milling (2014), Analytic expressions for ULF wave radiation belt radial diffusion coefficients, *J. Geophys. Res. Space Physics*, *119*, 251–258, doi:10.1002/2013JA019204.

Roberts, C. S., and M. Schulz (1968), Bounce resonant scattering of particles trapped in the Earth's magnetic field, *J. Geophys. Res.*, *73*(23), 7361–7376, doi:10.1029/JA073i023p07361.

Saikin, A. A., J.-C. Zhang, R. C. Allen, C. W. Smith, L. M. Kistler, H. E. Spence, R. B. Torbert, C. A. Kletzing, and V. K. Jordanova (2015), The occurrence and wave properties of H+, He+, and O+ band EMIC waves observed by the Van Allen Probes, *J. Geophys. Res. Space Physics*, *120*, 7477–7492, doi:10.1002/2015JA021358.

Santolík, O., D. A. Gurnett, J. S. Pickett, J. Chum, and N. Cornilleau-Wehrlin (2009), Oblique propagation of whistler mode waves in the chorus source region, *J. Geophys. Res.*, *114*, A00F03, doi:10.1029/2009JA014586.

Santolík, O., J. S. Pickett, D. A. Gurnett, J. D. Menietti, B. T. Tsurutani, and O. Verkhoglyadova (2010), Survey of Poynting flux of whistler mode chorus in the outer zone, *J. Geophys. Res.*, *115*, A00F13, doi:10.1029/2009JA014925.

Santolík, O., C. A. Kletzing, W. S. Kurth, G. B. Hospodarsky, and S. R. Bounds (2014), Fine structure of large-amplitude chorus wave packets, *Geophys. Res. Lett.*, *41*, 293–299, doi:10.1002/2013GL058889.

Schulz, M., and L. J. Lanzerotti (1974), *Particle Diffusion in the Radiation Belts*, Springer, New York.

Sheeley, B. W., M. B. Moldwin, H. K. Rassoul, and R. R. Anderson (2001), An empirical plasmasphere and trough density model: CRRES observations, *J. Geophys. Res.*, *106*, 25,631–25,642, doi:10.1029/2000JA000286.

Shklyar, D., and H. Matsumoto (2009), Oblique whistler-mode waves in the inhomogeneous magnetospheric plasma: Resonant interactions with energetic charged particles, *Surv. Geophys.*, *30*, 55–104, doi:10.1007/s10712-009-9061-7.

Shprits, Y. Y. (2009), Potential waves for pitch-angle scattering of near-equatorially mirroring energetic electrons due to the violation of the second adiabatic invariant, *Geophys. Res. Lett.*, *36*, L12106, doi:10.1029/2009GL038322.

Shprits, Y. Y., S. R. Elkington, N. P. Meredith, and D. A. Subbotin (2008a), Review of modeling of losses and sources of relativistic electrons in the outer radiation belt I: Radial transport, *J. Atmos. Sol. Ter. Phys.*, *70*, 1679–1693, doi:10.1016/j.jastp.2008.06.008.

Shprits, Y. Y., D. A. Subbotin, N. P. Meredith, and S. R. Elkington (2008b), Review of modeling of losses and sources of relativistic electrons in the outer radiation belt II: Local acceleration and loss, *J. Atmos. Sol. Ter. Phys.*, *70*, 1694–1713, doi:10.1016/j.jastp.2008.06.014.

Sibeck, D., R. W. McEntire, A. T. Y. Lui, R. E. Lopez, and S. M. Krimigis (1987), Magnetic field drift shell splitting: Cause of unusual dayside particle pitch angle distributions during storms and substorms, *J. Geophys. Res.*, *92*, 13,485–13,497, doi:10.1029/JA092iA12p13485.

Summers, D., and R. M. Thorne (2003), Relativistic electron pitch angle scattering by electromagnetic ion cyclotron waves during geomagnetic storms, *J. Geophys. Res.*, *108*(A4), 1143, doi:10.1029/2002JA009489.

Tao, X., and J. Bortnik (2010), Nonlinear interactions between relativistic radiation belt electrons and oblique whistler mode waves, *Nonlinear Processes Geophys.*, *17*, 599–604, doi:10.5194/npg-17-599-2010.

Taubenschuss, U., Y. V. Khotyaintsev, O. Santolík, A. Vaivads, C. M. Cully, O. Le Contel, and V. Angelopoulos (2014), Wave normal angles of whistler-mode chorus rising and falling tones, *J. Geophys. Res. Space Physics*, *119*, 9567–9578, doi:10.1002/2014JA020575.

Thorne, R. M. (2010), Radiation belt dynamics: The importance of wave-particle interactions, *Geophys. Res. Lett.*, *372*, L22107, doi:10.1029/2010GL044990.

Tian, T., Q. G. Zong, Z. Chang, Y. F. Wang, and X. Yang (2015), Statistical analysis of one Chinese sun-synchronous satellite anomalies, *Sci. China Tech. Sci.*, *59*, 540–546, doi:10.1007/s11431-015-5917-x.

Tsyganenko, N. A., and M. I. Sitnov (2005), Modeling the dynamics of the inner magnetosphere during strong geomagnetic storms, *J. Geophys. Res.*, *110*, A03208, doi:10.1029/2004JA010798.

Turner, D. L., S. K. Morley, Y. Miyoshi, B. Ni, and C.-L. Huang (2012), Outer radiation belt flux dropouts: Current understanding and unresolved questions, in *Dynamics of Earths Radiation Belts and Inner Magnetosphere, Geophys. Monogr. Ser.*, vol. 199, edited by D. Summers et al., pp. 195–212, AGU, Washington, D. C., doi:10.1029/2012GM001310.

Turner, D. L., et al. (2014a), On the cause and extent of outer radiation belt losses during the 30 September 2012 dropout event, *J. Geophys. Res. Space Physics*, *119*, 1530–1540, doi:10.1002/2013JA019446.

Turner, D. L., et al. (2014b), Competing source and loss mechanisms due to wave-particle interactions in Earth's outer radiation belt during the 30 September to 3 October 2012 geomagnetic storm, *J. Geophys. Res.*, *119*, 1960–1979, doi:10.1002/2014JA019770.

Ukhorskiy, A. Y., M. I. Sitnov, R. M. Millan, B. T. Kress, J. F. Fennell, S. G. Claudepierre, and R. J. Barnes (2015), Global storm time depletion of the outer electron belt, *J. Geophys. Res. Space Physics*, *120*, 2543–2556, doi:10.1002/2014JA020645.

Verkhoglyadova, O. P., B. T. Tsurutani, and G. S. Lakhina (2010), Properties of obliquely propagating chorus, *J. Geophys. Res.*, *115*, A00F19, doi:10.1029/2009JA014809.

Zhao, H., X. Li, J. B. Blake, J. F. Fennell, S. G. Claudepierre, D. N. Baker, A. N. Jaynes, D. M. Malaspina, and S. G. Kanekal (2014), Peculiar pitch angle distribution of relativistic electrons in the inner radiation belt and slot region, *Geophys. Res. Lett.*, *41*, 2250–2257, doi:10.1002/2014GL059725.